THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XIX — JANUARY 1904 — NUMBER 1

THE BLACK BODY AND THE MEASUREMENT OF EXTREME TEMPERATURES.[1]

By ARTHUR L. DAY and C. E. VAN ORSTRAND.

W. MICHELSON, in a recent address published in the *Journal of the Russian Physico-Chemical Society* of St. Petersburg,[2] says:

You are all aware, of course, that the Carnot-Clausius principle, that is to say, the so-called second law of thermodynamics, is one of the most general and most fruitful laws of contemporary physics. But it is also one of the most difficult to understand and to master in the concrete. Every series of arguments connecting this law either with experimental data or with equations of analytical mechanics, or, finally, with the principles of the theory of probabilities—in every case the light falls only on one side of the law, and no formula has yet been found to express completely its full scope.

[1] Read before the Philosophical Society of Washington, November 7, 1903, by the authors.

[2] Vol. XXXIV, No. 5, pp. 155–201, 1902 (Russian). It is very unfortunate that this paper has not been translated into some one of the more familiar languages of scientific literature. W. Michelson was the first to attempt to formulate a general equation for the distribution of energy in the continuous spectrum, and his critical examination of the steps by which the study of ether thermodynamics has advanced in the fifteen years since that time is itself a splendid contribution to the progress of the science. The authors are indebted to this paper all through the present discussion, and only regret that limits of space and title prevent the reproduction of considerable portions of it *in extenso*. The extracts here reproduced were translated by MR. ROBERT STEIN, of the U. S. Geological Survey.

The problem in ether thermodynamics offered by the spectrum is singularly analogous to the mechanics of a heated body. If we were able to follow out every detail of the motion of a single molecule of a heated body, we should undoubtedly find it conforming to the differential equation of Lagrange. And if we could select out a particular train of polarized, monochromatic oscillations from some spectrum, it would probably be completely expressed by Maxwell's differential equation. There is no reason to doubt that these solutions fully interpret the elementary phenomena as far as our present opportunities for verification extend. And yet an actual aggregate of such heated molecules, like a bundle of simple oscillations, in a very short period of time becomes so complicated that the integrating constants are hopelessly varied and distributed. There is no recourse but to statistical methods and average magnitudes. The unordered character of the processes occurring in both mechanical and electromagnetic phenomena is not due to infringement of the elementary differential equations of the one science or the other, but solely to the enormous complexity of the phenomena.

Tentative advances and partial solutions of the problem of radiation have been made in considerable number, greatly stimulated by the construction recently of an experimental black body capable of such exact manipulation as to outstrip, for the moment, the analytical advance. The steps in this development have almost all been contributed from abroad, and they have followed each other with such rapidity that those of us who have been compelled to follow their progress from a distance have hardly been able to see what has stood and what has fallen in the keen contest between the mathematical and experimental development. The obvious importance of any generalization which will enable us to determine the thermal condition of a remote body (the temperature of the Sun, for example), or one so hot that our usual laboratory apparatus cannot approach it without courting destruction, will be sufficient ground for scrutinizing the theories which have been propounded, examining carefully in how far they are supported by experimental data, and with what degree of probability they can be extrapolated to

extreme temperatures to meet the present needs of several branches of science.

The "black body"[1] lies at the very foundation of the quantitative study of radiant energy. It was first defined by Kirchhoff[2] in 1860, as a body which absorbs *all* the radiation which falls upon it—which reflects none and transmits none.[3] The experimental black body did not come until thirty-five years later (Wien and Lummer, 1895),[4] although Kirchhoff himself stated the conditions essential to its physical realization very explicitly in his original memoir. A radiation identical with that which an ideal black surface would emit, he says, is realized within every inclosed cavity in an opaque body, all parts of which have the same temperature. The properties of this radiation do not depend upon the properties of the walls forming the inner surface of the cavity, provided the latter have a finite light-emitting power for every wave-length (*i. e.*, are not perfect reflectors). Messrs. Wien and Lummer therefore simply prepared a hollow sphere, coated inside with lamp-black for low temperatures and oxide of iron or uranium for the higher temperatures, and provided with a single small opening. The principle will appear at once from the figure (Fig. 1). So much of the radiant energy as is not absorbed where it first impinges (a) upon the surface, must meet it again and again until the last surviving increment disappears.[5] No surface has ever been found which fulfils Kirchhoff's definition of blackness.

[1] The name "black body" is now in general use to describe a ***perfectly absorbing*** body. It begins to emit light when the temperature reaches about 525° C. The terms "black body" and "black radiation" are therefore somewhat misleading at the higher temperatures.

[2] G. KIRCHHOFF, "Ueber das Verhältniss zwischen dem Emissionsvermögen und dem Absorptionsvermögen der Körper für Wärme und Licht," *Pogg. Ann.*, **109**, 275, 1860.

[3] " . . . . welcher alle Strahlen, die auf ihn fallen, vollkommen absorbirt, also Strahlen weder reflektirt noch hindurchlässt."

[4] W. WIEN and O. LUMMER, "Methode zur Prüfung des Strahlungsgesetzes absolut schwarzer Körper," *Wied. Ann.*, **56**, 451, 1895.

[5] Strictly speaking, some rays will escape through the opening, however small. This of course involves a small error, but it is of negligible magnitude.

The part which the black body plays in the study of radiation will appear in a moment. In the paper above referred to, Kirchhoff fully established the first great law[1] in the thermodynamics of the ether, or, perhaps more strictly speaking, he was the first to make a general application of the second law of thermodynamics to radiation. He states it in this way: The ratio between the radiating and absorbing power for a given wave-length and temperature is the same for all bodies, and must therefore be a universal function of temperature and wave-length.[2] Thus for every substance emitting purely calorific radiation,

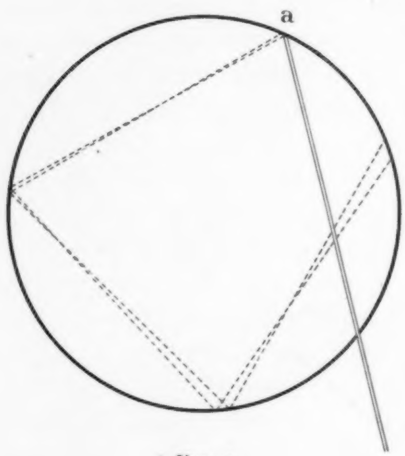


FIG. 1.

$$\frac{i_\lambda}{a_\lambda} = e_\lambda = f(\lambda, T) \ ,$$

i_λ = radiating power for wave-length λ,
a_λ = absorbing power for wave-length λ,
e_λ = radiating power for wave-length λ of the black body;

and for total radiation,

$$\frac{I}{A} = E.$$

Suppose we write the latter relation

$$\frac{I}{E} = A,$$

I is the energy of the total radiation from a given body, E the same function for an ideal, perfect radiator or black body

[1] An excellent demonstration of Kirchhoff's law will be found in DRUDE'S *Theory of Optics*, English translation, Longmans, Green & Co., 1902, p. 496.

[2] "Das Verhältniss von Emissions- und Absorptionsvermögen, bezogen auf die gleiche Wellenlänge und die gleiche Temperatur, ist für alle Körper das gleiche."

(Kirchhoff's definition), $\frac{I}{E}$ is then simply the number which represents the radiating power of the given body in terms of its unit, the perfect radiator, and is numerically equal to the absorbing power (A), which may be defined as the ratio between the quantity of radiant energy retained by the body and the total amount received. The function of the black body is therefore clear; it serves as the standard or unit of measurement which makes out of the study of radiation a quantitative science.

e_λ (energy radiated by a black body for the wave-length λ) is frequently called "Kirchhoff's function." He did not define the form of the function, but reasoned that it must be a simple one, being the same for all bodies, and therefore a natural law. He believed that the fundamental laws of nature were all simple.

Clausius,[1] who first formulated the second law of thermodynamics, was also greatly interested in its application to radiation. He proved that the Kirchhoff relation could not be independent of the surrounding medium without violating the second law, and finally that no phenomenon of heat radiation can produce motion in contradiction to the second law. This paper appeared in 1864 and, with the classical paper of Kirchhoff, forms the sound basis of a rapidly developing science.

In developing the electromagnetic theory of light, Maxwell[2] found that one of the consequences of his analysis was a resultant pressure in the direction of transmission, numerically equal to the energy per unit of its volume. Although the force was very small, Maxwell thought that a suitable light body suspended in a vacuum ought to show it. Several of these attempts were made at that time, but, owing to the relatively great disturbances in the residual gas, the phenomenon could not be experimentally established. The discovery was of considerable significance to astronomy, but no immediate application to physical problems appeared.

[1] R. CLAUSIUS, "Die Concentration von Wärme- u. Lichtstrahlen u. die Grenzen ihrer Wirkung," *Pogg. Ann.*, **121**, 1–44, 1864.

[2] *Electricity and Magnetism*, chapter on "Electromagnetic Theory of Light."

Three years later, in 1876, Bartoli[1] arrived at the conclusion that light waves must exert a pressure in the direction of their propagation, by a process of thermodynamic reasoning. His line of argument is entirely different from Maxwell's, and is approximately this: To suppose equilibrium between energy of radiation and the temperature, it is necessary to assume a synonymous functional dependence between the latter and the energy per unit volume. If this density be increased by diminishing the volume while T remains unchanged, energy must necessarily be communicated to the adjacent walls of the vessel so long as the equilibrium is not re-established. His system is reviewed by W. Michelson[2] as follows:

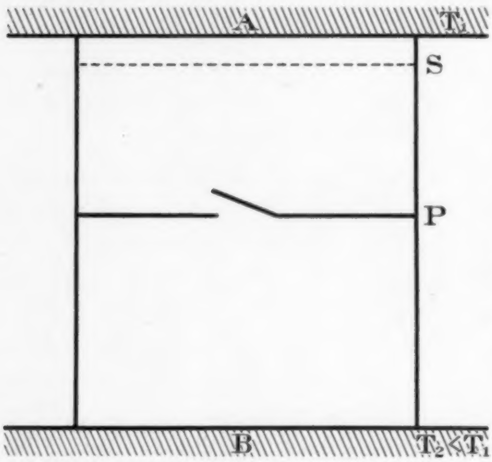


FIG. 2.

Imagine two perfectly black bodies A and B (Fig. 2), having temperatures T_1 and $T_2 < T_1$, to include between them a perfectly white (diffusely reflecting) cylinder equipped with a piston P and a screen S which may be used to screen A, and both impervious to radiation. Lower the piston with the valve open, slip the screen in front of A, then raise the piston with the valve closed and withdraw S. It is plain that any amount of heat in the form of diffuse radiant energy can be transferred in this way from the colder body B to the warmer body A. According to the second law, this process is only possible when work is done, which in this case must be sought in the raising of the piston

[1] BARTOLI, *Sopra i movimenti prodotti dalla luce e dal calore.* Le Monnier, Florence, 1876. See also *Wied. Ann.*, **22**, 31, 1884. A third proof is given by GALITZINE, *Wied. Ann.*, **47**, 479–495, 1892; *Phil. Mag.*, (5) **35**, 113–126, 1893, and a fourth by H. PELLAT, *Journ. d. Phys.* for July 1903.

[2] *Loc. cit.*, 1902.

with the valve closed. And this work will suffice as compensation for the transfer of the energy only in case the diffuse radiation exerts a pressure on the piston which is proportional to its density. Thus purely thermodynamic reasoning has led to the same conclusion which Maxwell reached in his electromagnetic theory of light. Bartoli also prepared elaborate experiments in an effort to verify his conclusion experimentally, but was unable to eliminate the disturbances due to the surrounding gas.

After another three-year interval, light was thrown upon the problem of radiation from a different source. Stefan[1] noticed that in some experiments of Tyndall upon the radiation from glowing platinum wire between the first red heat (525° C.) and full white heat (estimated at 1200° C.), the radiated energy was proportional to the fourth power of the absolute temperature. He verified this relation (which bears his name) upon existing experimental data (Dulong and Petit, de la Prevostaye and Desains, and others) and published it as the conclusion toward which all the data gathered up to that time clearly pointed. This law plainly asserted, without qualification, that for a given temperature the quantity of radiant energy emitted by all solid bodies was the same, although its spectral composition was known to differ widely. Apparent support was found for this in the very generally accepted earlier law of Draper (1847),[2] that red heat begins in all solid bodies at the same temperature (525° C.). Recently more exact methods and measurements[3] have shown that neither of these generalizations is more than roughly true. Different bodies have been found to possess different radiating powers and to reach red heat at different temperatures.[4]

It remained for Boltzmann[5] in 1884 to bring these threads

[1] J. STEFAN, "Ueber die Beziehung zwischen der Wärmestrahlung und der Temperatur," *Wien. Akad. Ber.*, II. Serie, **79**, II. Abth., 391, 1879.

[2] J. W. DRAPER, *Amer. Jour. Sci.*, (2) **4**, 1847; *Phil. Mag.*, (3) **30**, 1847; *Scientific Memoirs*, London, 44, 1878.

[3] H. F. WEBER, EMDEN, LUMMER and associates, BOLTZMANN (theoretical considerations), and others.

[4] No body can reach red heat *before* the black body.

[5] L. BOLTZMANN, "Ableitung des Stefan'schen Gesetzes, u.s.w. aus der electromagnetischen Lichttheorie," *Wied. Ann.*, **22**, 31 and 294, 1884.

together and to interpret the real meaning of Stefan's law with the help of the light-pressure which Maxwell and Bartoli, each from a different standpoint, had reasoned must exist, but of which they were not able to find experimental evidence. Boltzmann imagines a cylinder (Fig. 3) similar to Bartoli's, but open at one end, in which a piston incloses radiant energy of the temperature of the heat reservoir (the black body *B* closing end of cylinder). He then interposes an impervious screen between the piston and reservoir, and allows the piston to move away from the fixed screen, producing a radiational adiabatic expansion. Now, in order to treat this under the second law of thermodynamics $\left(\int \frac{dQ}{T} = 0\right)$, the process had to be reversible, and to make it reversible it was necessary to assume that for identical values of the total density of the radiation its *wave-length composition* will always be identical, irrespective of the manner in which the change of density proceeds — whether by fall of temperature or by adiabatic expansion. This assumption, which is most important for the demonstration, was not proved until nine years later by W. Wien.[1] Boltzmann then proceeded, after a manner closely analogous to the demonstration of the resolution of pressures along the co-ordinate axes in a perfect gas, to prove that the pressure of completely diffuse radiation on the surface of the piston is exactly equal to one-third of the density. From these

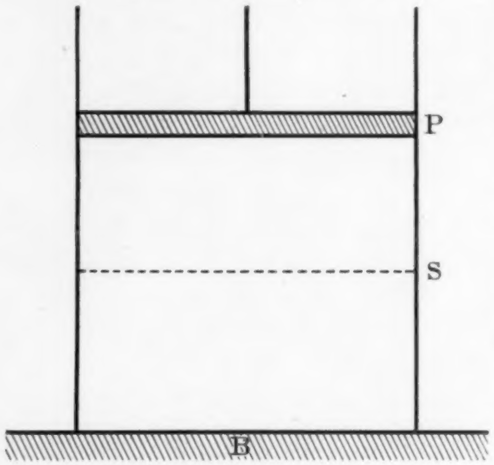


FIG. 3.

[1] W. WIEN, "Eine neue Beziehung der Strahlung schwarzer Körper zum zweiten Hauptsatz der Wärmetheorie," *Sitzungsber. Berl. Akad.*, **6**, 55–62, 1893.

two important theorems, a few steps of simple formulation brought him to the so-called Stefan law,[1]

$$E = \int_0^\infty e_\lambda \, d\lambda = \sigma T^4 ,$$

but with an important difference. Stefan promulgated his relation as true of all bodies without regard to the quality of the radiation (color); Boltzmann deduces it only for *complete* or *black radiation.* The Stefan-Boltzmann law as finally established (1884) may be stated in words as follows: The energy of completely diffuse (black) radiation is directly proportional to the fourth power of the absolute temperature.

It only remains to add the evidence bearing upon the law, which has been gathered in the interval since the black body

[1] The derivation is given briefly by W. WIEN in "Les lois théoriques du rayonnement," *Rapport, congrès international de physique*, **2**, 20, as follows:

$$dQ = dU + dW.$$

dQ = differential of the radiant heat entering the cylinder.
dU = increase of internal energy.
dW = external work.

$$u = v\psi.$$

v = volume.
ψ = density of energy (intensity per unit volume).

$$dW = p dv = \frac{1}{3} \psi dv \text{ (Boltzmann).}$$

$$dQ = d(v\psi) + \frac{1}{3}\psi dv = v d\psi + \frac{4}{3}\psi dv.$$

An integrating factor of the right-hand member is $\psi^{-\frac{1}{4}}$, of the left, $\frac{1}{T}$, hence

$$T = b\sqrt[4]{\psi} , \ \psi = \frac{T^4}{b^4} = \sigma T^4 \text{ (Stefan Boltzmann law).}$$

The differential of the entropy is

$$dS = \frac{dQ}{T} = \frac{v}{b}\psi^{-\frac{1}{4}} d\psi + \frac{4}{3b}\psi^{\frac{3}{4}} \, dv = d\left[\frac{4}{3b} v\psi^{\frac{3}{4}}\right].$$

Integrating

$$S = \frac{4}{3b} v\psi^{\frac{3}{4}} + \text{const.}$$

In adiabatic expansion

$$\psi v^{\frac{4}{3}} = \text{const.},$$

which is exactly analogous to Poisson's

$$pv^k = \text{const.}$$

in the thermodynamics of gases. The relation of the two heat capacities is supplied by the number $\frac{4}{3}$ as a direct consequence of the fact that the light-pressure of black radiation is equal to $\frac{1}{3}$ of its density.

became a laboratory instrument (1895). A single table taken from Professor Lummer's measurements[1] will serve to show the character of the experimental verification:

Absolute Temperature Observed	σ	Absolute Temperature Calculated	T.Obs. – T. Calc.
373°.1	127.0	374°.6	−1°.5
492.5	124.0	492.0	+0.5
723.0	124.8	724.3	−1.3
745.0	126.6	749.1	−4.1
810.0	121.6	806.5	+3.5
868.0	123.3	867.1	+0.9
1378.0	124.2	1379.0	−1.0
1470.0	123.1	1468.0	+2.0
1497.0	120.9	1488.0	+9.0
1535.0	122.3	1531.0	+4.0
Mean	123.8		

The observed temperatures are in terms of the Holborn-Day gas thermometer scale,[2] extrapolated above 1150° C. with the help of platinum—platin-rhodium thermo-elements.[3]

The last two years have also witnessed the experimental verification of the Maxwell-Bartoli light-pressure theorem. Lebedew[4] in 1901 not only succeeded in showing the existence of a pressure in the direction of propagation of light, but measured it and found it to be within about 20 per cent. of the value anticipated in the Maxwell-Bartoli analyses. Quite independently of Lebedew, Nichols and Hull[5] have been at work upon the same

[1] See O. LUMMER, "Le rayonnement des corps noirs," *Rapports congr. intern. de phys.*, **2**, 41 (Gauthier-Villars, Paris, 1900), where the full literature will be found.

[2] LUDWIG HOLBORN and ARTHUR L. DAY, "Ueber das Luftthermometer bei hohen Temperaturen," *Ann. der Phys.*, **2**, 505, 1900; Engl. transl., *Am. Jour. Sci.*, (4) **10**, 171, 1900.

[3] LUDWIG HOLBORN and ARTHUR L. DAY, "Ueber Thermoelektricität einiger Metalle," *Sitzungsber. Berl. Akad.*, 691, 1899; Engl. transl., *Am. Jour. Sci.*, (4) **8**, 303, 1899.

[4] P. LEBEDEW, *Ann. d. Phys.*, **6**, 433–458, 1901; *Rapports congrès international*, Paris, 1900, 133 *seq.*

[5] E. F. NICHOLS and G. F. HULL, preliminary publication, *Phys. Rev.*, November 1901; final results, July and August 1903; ASTROPHYSICAL JOURNAL, **17**, 315–351, June 1903.

problem, and with such refinement of method that they have just been able to announce perfect agreement with the theoretically established value within the limits of their observation errors (about 1 per cent). Their observations include measurements upon light passing through air of various pressures, through red glass and through water.

If this review has been clear, we now see that this important general law of ether thermodynamics boasts exceptionally complete theoretical development, supported by experimental data both in its main hypothesis and in its conclusion. It is therefore very generally regarded as established.

The law of Stefan-Boltzmann gives us the total energy of the spectrum of the black body, but says nothing in regard to the energies of the component monochromatic radiations (e_λ, Kirchhoff's function), their relation to temperature, and their distribution in the spectrum. Even before this time attempts to solve the distribution of the energy in the spectra of various bodies had been made,[1] but the first particularly noteworthy effort was due to W. Michelson[2] in 1887. In the previous year S. P. Langley[3] had published some observations upon the general features of the energy curves obtained from the Sun, the arc light, and some other sources, and from these Michelson endeavored to reach a general theory. His working hypothesis was that the continuity of the spectrum emitted by solid bodies could be explained only by complete disorder in the vibrations of their atoms, and that each periodic change in the motion of an atom produces a wave of the same period in the surrounding ether. Then he assumed that Maxwell's law of the most probable distribution of velocities among a great number of molecules is valid for solids, and, by very ingenious reasoning, based

[1] The distribution of energy in the spectra of particular solid bodies and of the Sun was measured much earlier (Langley) and received some theoretical consideration (E. v. LOMMEL, *Wied. Ann.*, **3**, 251, 1877).

[2] "Essai théorique sur la distribution de l'énergie dans les spectres des solides," *Journ. de la Soc. Phys., Chim. russe*, (4) **19**, 79, 1887; *Journ. de Phys.*, (2) **3**, 467, 1887.

[3] S. P. LANGLEY, *Ann. d. Chim. et Phys.*, (6) **9**, 433–506, 1886.

largely upon the theory of probabilities,[1] deduced a general equation for the black body:

$$E_\lambda = c_1 T^{\frac{3}{2}} f(T) \lambda^{-(2p+4)} e^{\frac{-c_2}{\lambda^2 T}}$$

Or, when combined with Stefan's law,

$$E = c_1 T^{\frac{3}{2}} \lambda^{-6} e^{\frac{-c_2}{\lambda^2 T}}$$

The apparent support of this deduction furnished by the Langley data proved to be illusory, as the Langley tables were obtained from prismatic spectra and did not apply to relations in the normal spectrum.

Other attempts quickly followed this, but they contributed little to the advancement of the subject, and, with perhaps one exception, are not of interest now. The condition for maximum radiation in Weber's equation (1888)[2] required that the wavelength for which the energy is a maximum should be shorter as the temperature increases:

$$\lambda_{\text{max.}} T = \text{const.},$$

i. e., with increasing temperature the greatest intensity of radiation approaches the visible spectrum. This appears to have been the first publication of a relation which is characteristic of the more important equations which have followed and which has received abundant independent verification from experiment.

The next really important attempt to derive a general equation to express the distribution of spectral energy, was made, after an interval of eight years, by W. Wien in 1896.[3]

Although this long interval produced no general distribution equation of importance, it is marked by two important steps toward it—the demonstration of the so-called "displacement law" by W. Wien (1893)[4] and the achievement of an experi-

[1] One point in this demonstration of probabilities, which here appears for the first time, is of more than incidental importance. Michelson supposes each particle to have a fixed position of equilibrium about which it constantly oscillates as in the usual theory of solid bodies. He then demonstrates from simple hypotheses that the most probable path of such particles approximates very closely to successive diameters of its sphere of activity.

[2] H. F. WEBER, *Sitzungsber. Berl. Akad.*, II, 933–957, 1888.

[3] W. WIEN, "Ueber die Energievertheilung im Emissionsspektrum des schwarzen Körpers," *Wied. Ann.*, **58**, 662, 1896.

[4] *Loc. cit.*

mental black body by Wien and Lummer (1895).[1] The latter represented little more than the practical carrying out of the directions of Kirchhoff in his original memoir, to which reference has already been made, but it marks an epoch in the study of radiation. It at once became possible to verify the general relations which had been developed in considerable number, and furnished initiative and direction to further analytical effort.

The Wien displacement equation had also been long desired and partly anticipated. Boltzmann had assumed, but not proved, that there was no difference in the *composition* of black radiation, whether its energy-density were increased by adiabatic compression or by increase of temperature. Wien explained this by an ingenious application of Doppler's principle to radiant heat. His reasoning is briefly this: If the radiation from a black body is contained within perfectly white (diffusely reflecting) walls, a diminution of volume can be accomplished only by doing work against the pressure of the radiation, and is accompanied by an increase in the density (energy per unit volume) of the radiation. The second law of thermodynamics requires that the spectral composition of the compressed radiation, be the same as though the increase in the density had been produced by increasing the temperature. Both theory (Boltzmann)[2] and experiment (Weber)[3] had shown that wave-lengths were shortened in a simple ratio by increasing the temperature. Wien demonstrated that compressing inclosed radiant energy in this way must involve Doppler's principle, and that this was necessary and sufficient to account for the shortening of all wave-lengths in the same ratio. The relation of color and temperature he found to be[4]

$$\frac{\lambda}{\lambda_0} = \frac{\theta_0}{\theta} \; ;$$

therefore, as Wien himself expresses it, "when the temperature increases, the wave-length of every monochromatic radiation diminishes in such a manner that the product of the temperature

[1] *Loc. cit.* [2] 1884, *loc. cit.* [3] 1888, *loc, cit.*

[4] Given in DRUDE'S *Theory of Optics*, Engl. trans., 516–522, 1902.

and the wave-length is constant." From this equation Weber's maximum relation,

$$\lambda_{max.} T = \text{const.} ,$$

appears in its proper context as a special case. And if we go farther, as Wien did, and combine the equation with the one which represents the Stefan-Boltzmann law, a new relation follows:

$$E = T^5 \phi(\lambda T) ,$$

in which the energy appears not merely as a general function of T and λ, but as a specific function of T and of the product λT. From this combination of the displacement law with that of Stefan, it is possible to calculate the distribution of energy in the spectrum of the black body for any temperature whatever, provided only that the distribution is known for a single temperature. Of course, this conclusion served to bend every effort to the determination of the energy — wave-length relation for a particular temperature, a problem to which ten full years have now been given without reaching a wholly satisfactory result.

Before proceeding to the discussion of the more important efforts in this direction, attention should be called to one weak link in the chain of reasoning thus far. Wien's combination of Stefan's law with his own displacement law involves assumptions which, without the practically unanimous support of experiment, must be open to criticism. How, says Michelson,[1] are we to apply the Stefan-Boltzmann law to *monochromatic* radiation, for which it has never been proved true? And again, what is the status of the term "temperature" when applied to free monochromatic radiation in the ether, which is not in equilibrium with any hot body? It is, indeed, a long analytical step from completely diffuse radiation in equilibrium with its envelope, to free monochromatic radiation. And yet the inferred analogy seems to be justified *a posteriori* by experiment. The expression

$$E_{max.} T^{-5} = \text{const.} ,$$

which is obtained by combining the Wien and Stefan laws, has been fully verified by the experiments of Lummer, Paschen,

[1] *Loc. cit.*, 1902.

Rubens, Pringsheim, Wanner, Kurlbaum, and others. A representative table of experimental values from Lummer[1] follows:

T Absolute	λ_m	E_m	$\lambda_m T$	$\frac{E_m}{T^5}$	T Computed	Difference Degrees
621°.2	4.53	2.026	2814	2190	621°.3	+0°.1
723.0	4.08	4.28	2950	2166	721.5	−1.5
908.5	3.28	13.66	2980	2208	910.1	+1.6
998.5	2.96	21.50	2956	2166	996.5	−2.0
1094.5	2.71	34.0	2966	2164	1092.3	−2.2
1259.0	2.35	68.8	2959	2176	1257.5	−1.5
1460.4	2.04	145.0	2979	2184	1460.0	−0.4
1646.0	1.78	270.6	2928	2246	1653.5	+7.5
		Mean	2940	2188		

Some recent observations by Paschen[2] establish the constant even more sharply:

T Absolute	λ_m	$\lambda_m T$	E_m	$\frac{E_m}{T^5 \times 10^{15}}$
876°.4	3.333	2914	0.637	1.232
875.0	3.328	2911	0.642	1.252
1052.2	2.781	2926	1.667	1.293
1053.7	2.770	2919	1.660	1.277
1055.2	2.777	2929	1.643	1.255
1337.4	2.188	2926	5.454	1.275
1347.4	2.172	2927	5.499	1.238
1352.7	2.159	2920	5.665	1.251
1357.4	2.157	2929	5.797	1.258
1567.9	1.858	2913	12.28	1.296
1580.6	1.846	2918	12.26	1.242
	Mean	2921		1.268

We may now return to the general law of distribution of energy and W. Wien's second important contribution (1896) to the study of radiation, for he was also the first to attack the main issue which he had so clearly pointed out with his displacement equation. Like Michelson, Wien draws heavily upon the theory of gases for analogies from which to formulate a general theory of radiation for the black body. No one has yet suc-

[1] *Rap. congr. int.*, **2**, 82.

[2] "Ueber das Strahlungsgesetz des schwarzen Körpers." *Ann. d. Phys.*, (4) **4**, 277, 1901.

ceeded in deducing a theory of spectral distribution from established generalizations alone. It has always been found necessary to postulate important special relations which must then be judged according to the accuracy with which quantitative experimental results are interpreted. Wien makes the following special hypotheses:[1]

1. In a gas which sends out radiant energy, each molecule emits only radiations of a single wave-length. The vibration period is a function of the molecular velocity only.

2. The intensity of the radiation included between two neighboring wave-lengths is proportional to the number of molecules which emit radiations of that particular period.

From these and the law of Maxwell, which Michelson had employed, together with his own displacement law, Wien developed a new equation:

$$\phi_\lambda d\lambda = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}} .$$

Compared with Michelson's treatment of the problem, two important differences appear in that of Wien—the return to thermodynamic reasoning throughout, and the use of the displacement law, which was demonstrated long after Michelson's paper appeared.

It will throw much light on the theoretical problem, and on the character and consequences of the arbitrary hypotheses of which Wien and others have been obliged to make use, if we quote briefly from Michelson's criticism of Wien's assumptions in the recent article referred to at the beginning of this paper, bearing in mind that six years of great theoretical and experimental activity had passed since Wien derived his law, and sixteen years since Michelson's own earlier publication:

. . . . But with the formulation of this law (Wien's), as given by the author himself and accepted, among others, by Planck, I do not entirely agree.

The point at issue is this, that Wien affirms that completely unordered (black) radiation consists of innumerable multitudes of different monochromatic radiations which, he says, are merely mixed mechanically and possess each its own individual energy; on varying the temperature, each by itself

[1] *Rap. congr. int.*, **2**, 35.

follows the law of Stefan, and besides, according to the "displacement law," changes its wave-length, λ, in inverse proportion to the absolute temperature. The condition of thermal (radiational) equilibrium W. Wien and Planck find in the *identical temperatures* attributed to all elementary monochromatic radiations. But to assert this is, in my opinion, to carry too far the analogy between black radiation and a mixture of a multitude of different gases.

In the first place, to speak of "temperature" of monochromatic radiation is possible only very conditionally and with reservations, because such radiation cannot be in thermal equilibrium with any actually hot body. In the second place, I hold it as certain that, in black radiation, the different monochromatic constituent parts of it have not each its own individual existence, but continually mingle their energies. If such an uninterrupted exchange of energy does not take place among the different radiations, I do not see how we can rationally explain the Kirchhoff function e_λ as synonymous with the dynamic stability of completely unordered radiation. . . . . In this respect, the diffuse radiation inclosed in an opaque envelope differs essentially from the radiation which is freely propagated in the ether. Here a determined period of oscillation is indissolubly connected with every element of energy, and the latter is completely characterized by just that period, together with a particular amplitude and azimuth of polarization. Upon removal of the source, and upon passing through the different centers, the amplitude and polarization may change; the period alone (except under a few rare conditions) remains unchanged. It then brings us information even from the different stellar worlds which give spectra of thin lines. But in the diffuse radiation of a black body there are no traces of any lines, there are no indissoluble ties connecting the period with the individual portions of radiant energy. . . . . Just as the heat equilibrium of gas, characterized by the law of Maxwell, is only maintained, because of the uninterrupted exchange of velocity among the different molecules of the gas, so also the law of Kirchhoff is verified by the uninterrupted exchange of energy between oscillations of different periods.

At first, after Wien's law appeared, there seemed to be good reason for believing that the problem was finally solved. A few weeks before its publication, a mass of new experimental data had been published by Paschen[1] for a number of substances closely approximating to the black body. To these the formula was at once applied and successfully. Planck[2] soon after succeeded in deducing the Wien formula from quite different

[1] "Ueber Gesetzmässigkeiten in den Spektren fester Körper," *Wied. Ann.*, **58**, 455, 1896.

[2] "Ueber irreversible Strahlungsvorgänge," etc., *Sitzungsber. Berl. Akad.*, 1897, 57, 715, 1122; 1898, 449, and 1899, 440; also *Ann. d. Phys.*, **1**, 69 and 719, 1900.

assumptions, using the electromagnetic theory of light, whereas Wien had depended chiefly upon thermodynamics. Meanwhile Paschen had continued his experimental measurements with an actual black body, and his results, contained in two papers,[1] agreed perfectly with the Wien-Planck law.

But Wien's law had come at a time (one year after the development of the experimental black body) when very exact experimental data had to be reckoned with, and so the twofold derivation of the law with the confirmatory experiments of Paschen did not suffice to establish it without a question.

The story is a most interesting one. First Professor Lummer, who with Wien himself built the first black body, was unable with a very effective plant (at the *Reichsanstalt*), to confirm Paschen's results, nor would his data fit the Wien-Planck equation. For the constants c_1 and c_2 of the formula he obtained:[2]

λ	$1.21\ \mu$	$1.96\ \mu$	$2.20\ \mu$	$3.63\ \mu$	$4.56\ \mu$
c_2	13510	13810	14240	14800	16510
$c_1 \cdot 10^{-11}$	1067	1219	1449	1771	2261

and later with Pringsheim,[3] over a greater range of wave-lengths:

λ	$1.21\ \mu$	$4.56\ \mu$	$8.3\ \mu$	$12.3\ \mu$	$17.9\ \mu$
c_2	13510	16510	18500	24800	31700

in which not only large but systematic variations from the constant value appear. Then Beckmann[4] made a similar determination for the wave-length[5] $\lambda = 24\,\mu$ and found $c_2 = 26{,}000$.

[1] "Ueber die Vertheilung der Energie im Spektrum des schwarzen Körpers bei niederen Temperaturen," *Berl. Akad. Ber.*, 405, 1899. "Ueber die Vertheilung der Energie im Spektrum des schwarzen Körpers bei höheren Temperaturen," *ibid.*, 959, 1899. ASTROPHYSICAL JOURNAL, **10**, 40, 1899; **11**, 288, 1900.

[2] *Rapports congr. international, loc. cit.* [3] *Verh. d. Deutsch. Phys. Ges.*, **2**, 1900.

[4] Inaugural Dissertation, Hannover, 1898.

[5] RUBENS has since shown (*Wied. Ann.* **69**, 576, 1899) that the substances used as sources of radiation by Beckmann must have yielded two wave-lengths, $\lambda = 24\,\mu$ and $\lambda = 32\,\mu$—which is immaterial in this connection however.

Then Lummer and Pringsheim[1] employed a sylvin prism and made further measurements between the wave-lengths 12 μ and 19 μ and the temperatures $-190°$ to $+1550°$, and found c_2 to vary with the temperature as well as with the wave-length. Still another careful set of measurements by Rubens and Kurlbaum,[2] extending to 50 μ, corroborated the foregoing (see tables, p. 22). And finally Paschen himself,[3] upon repeating his experiments, found that he was able to represent only the short wave-lengths by the Wien-Planck equation.

Notwithstanding this pressure of adverse experimental evidence and the somewhat strained theoretical reasoning, the Wien formula is difficult to displace. Both in its hypotheses and in its formulation it is the simplest and most straightforward of the expressions which have been proposed to represent the distribution of spectral energy in black radiation. In that part of the spectrum in which the maximum intensity occurs, and for a sufficient distance on either side of it to include by far the greater part of the total radiation expressed by Stefan's law, the Wien equation represents the distribution of the intensity exceedingly well. It is among the longer waves of the infra-red, from 6 μ on as far as measurements have been made (50 μ), that the formula totally fails.[4] It therefore offers no hope of successful extrapolation to inaccessible temperatures.

Equations of similar character to that of Wien have been developed by Thiesen,[5] Rayleigh,[6] Jahnke,[7] and others, to which some reference will be made farther on. They represent efforts to reproduce the experimental results more faithfully than the

[1] O. Lummer and E. Pringsheim, *Zeitschrift für Instrumentenkunde*, **20**, 149, 1900; *Verh. Deutsch. Phys. Ges.*, February 1900.

[2] "Ueber die Emission langwelliger Wärmestrahlen durch den schwarzen Körper bei verschiedenen Temperaturen," *Sitzungsber, Berl. Akad.*, 929–941, 1900; *Ann. d. Phys.*, **4**, 649–666, 1901.

[3] *Verh. d. Deutsch. Phys. Ges.*, **2**, 202, 1900, footnote; *Ann. d. Phys.*, **4**, 277–298, 1901.

[4] See tables, p. 22.

[5] "Ueber das Gesetz der schwarzen Strahlung," *Verh. d. Deutsch. Phys. Ges.*, **2**, 65–70, 1900.

[6] *Phil. Mag.*, **49**, 539, 1900.

[7] *Ann. d. Phys.*, **3**, 283–297, 1900.

Wien formula was found to do, by more or less empirical modifications of it. They may therefore be regarded as nearer approximations than Wien's in some cases, and possess more accessible constants in some, but they follow the same reasoning and cannot be said to have contributed anything further to the final solution of the problem.

Planck has a somewhat different method of approaching the subject of the distribution of energy in the radiation of the black body. It is hardly possible even to sketch the plan of his elaborate structure within the limits of this brief examination of the subject. He starts out in his first paper[1] with Maxwell's equation for the propagation of electromagnetic oscillations in the ether, and establishes the conditions for perfectly diffused rectilinear light. He then imagines an inclosed space filled with such disordered radiation to contain a great number of small, ideal, electric resonators, which are then supposed not to interfere with one another. Each resonator acquires some final oscillatory state depending only upon the intensity of the radiation of its particular period which falls upon it. For the monochromatic oscillations of each period he assumes the existence of a great number of independent resonators of the same period. He assumes further that within each such system of resonators and electromagnetic waves of a particular period, notwithstanding incomplete and irregular intermingling of phase, amplitude and azimuth of polarization and direction of light, there is established some stationary state depending on the character of the total energy of the given period. Out of this artificial system Planck has evolved two theories. In the first[2] he determined arbitrarily the entropy of such an ideal resonator in a field of completely diffused radiation, and obtained the same formula which W. Wien had deduced from thermodynamic considerations. He has now abandoned this theory. In the second[3] he follows certain ideas of Boltzmann's on the probability of recurrence of certain states

[1] "Irreversible Strahlungsvorgänge," *loc. cit.*, also *Ann. d. Phys.*, **1**, 69, 1900; "Entropie u. Temperatur strahlender Warme," *ibid.*, 719.

[2] 1897, *loc. cit.*

[3] "Ueber eine Verbesserung der Wien'schen Spektralgleichung," *Verh. d. Deutsch Phys. Ges.*, **2**, 202, 1900; *Sitzungsber. Berl. Akad.*, 544, 1901.

in an assumed system of atoms or molecules, and defines the entropy of one of the resonators by analogy between this assumed system and his own system of electromagnetic oscillations.

It is scarcely possible at this time to offer a fair criticism of Professor Planck's result. His formula fits all the more reliable observations which have been made over the entire range of accessible wave-lengths with unexceptionable exactness (see table below), but he frankly states that his definition of entropy is only one of an infinite number of possible definitions in such a system of electromagnetic oscillations, and that it was chosen with the experimental observations before him, because it was the simplest one which fitted. His definition is therefore in a measure "after the fact," and inasmuch as it is built upon a strongly ideal and exceedingly intricate system, it has not, so far, been regarded as possessing the essential characteristics of a final solution. Michelson, in the critical article before referred to, accepts it provisionally, though not with confidence. "The future only can show," he says, "whether it is merely a new degree of approximation which in time will meet the same fate that has overtaken the earlier spectral formulæ." Wien deprecates the extraordinarily complicated character of the reasoning and says it should be greatly simplified. It should be added that Planck has compared his expression for "electromagnetic entropy" with Boltzmann's determination of the entropy of a gas, and drawn some interesting inferences upon the masses of molecules and atoms, and on the magnitude of the charge carried by one electron.[1] The order of the quantities obtained is the same as obtained by the other methods.

A comparative table from Rubens and Kurlbaum[2] shows very clearly the superiority of the Planck formula in representing the experimental data for the longer wave-lengths:

[1] M. PLANCK, "Ueber d. Elementarquanta d. Materie u. d. Electricität," ***Ann. d. Phys.***, (4) **4**, 564–566, 1901.

[2] "Anwendung der Methode der Reststrahlen zur Prüfung des Strahlungsgesetzes," ***Ann. d. Phys.***, (4) **4**, 649, 1901; also ASTROPHYSICAL JOURNAL, **14**, 335, 1901.

TABLE I.

Absolute Temperature T	E Obs.	E Wien	E Rayleigh	E Planck
		$\lambda=8.85\ \mu$.		
0°	....	$-$ 1.96	$-$ 1.00	$-$ 1.41
100	$-$ 1.6	$-$ 1.96	$-$ 1.00	$-$ 1.41
200	$-$ 1.5	$-$ 1.82	$-$ 0.92	$-$ 1.31
300	....	$\pm$ 0.10	+ 0.10	+ 0.1
373	+ 3.4	+ 4.07	+ 2.21	+ 3.0
500	+ 13.5	+ 16.5	+ 9.60	+ 12.4
800	+ 53.5	+ 60.5	+ 44.3	+ 50.3
1100	+102.0	+107.0	+ 96.7	+ 99.8
1273	+132.0	+132.0	+132.0	+132.0
1400	+154.0	+147.7	+160.0	+154.6
1700	+212.5	+182.3	+229.0	+213.5
		$\lambda=24.0\ \mu$ and $31.6\ \mu$.		
0	....	$-$ 42.4	$-$ 10.7	$-$ 15.4
85	$-$ 15.5	$-$ 41.0	$-$ 10.5	$-$ 15.0
193	$-$ 9.4	$-$ 26.8	$-$ 7.4	$-$ 9.3
293	0	0	0	0
523	+ 30.3	+ 50.6	+ 25.3	+ 28.8
773	+ 64.3	+ 88.9	+ 58.3	+ 62.5
1023	+ 98.3	+114.5	+ 94.4	+ 96.7
1273	+132.0	+132.0	+132.0	+132.0
1523	+167.0	+145.0	+174.5	+167.5
1773	+201.5	+155.0	+209.0	+202.0
∞	....	+226.0	+ ∞	+ ∞
		$\lambda=51.2\ \mu$.		
0	....	$-$121.5	$-$ 20.0	$-$ 23.8
85	$-$ 20.6	$-$107.5	$-$ 19.0	$-$ 21.9
193	$-$ 11.8	$-$ 48.0	$-$ 11.5	$-$ 12.0
293	0	0	0	0
523	+ 31.0	+ 63.5	+ 28.5	+ 30.4
773	+ 64.5	+ 96.0	+ 62.5	+ 63.8
1023	+ 98.1	+118.0	+ 97.0	+ 97.2
1273	+132.0	+132.0	+132.0	+132.0
1523	+164.5	+141.0	+167.0	+166.0
1773	+196.8	+147.5	+202.0	+200.0
∞	....	+194.0		

The discussion of the title of Planck's second formula to be regarded as an adequate and final solution of the problem of spectral energy distribution may properly be left to the future. It represents all the trustworthy experimental observations of record, to the highest temperatures at which it has been found

possible to measure (1550° C.) and is the only one of the spectral distribution equations which has not yet been shown to be defective in its theory, or its application, or both.

To summarize, then, there are four quantitative relations defining the radiation from the black body, which are founded upon generally accepted theoretical reasoning; their constants are accessible to experiment and have been established within reasonable limits of error by all the leading experimental scientists who have contributed to the problem thus far:

(1) The Stefan-Boltzmann relation,

$$E = \int_0^\infty e_\lambda \, d\lambda = \sigma T^4 ,$$

representing the total radiant energy emitted by a black body as a simple function of the temperature.

(2) and (3) Two relations consequent upon W. Wien's theorem that with increasing temperature all wave-lengths are shortened in inverse ratio to the absolute temperature:

$$2. \quad \lambda_{max.} T = \text{const.}$$

$$3. \quad E_{max.} T^{-5} = \text{const.}$$

The same relations also define the conditions at the energy maximum in the distribution formulæ of Planck, Wien, and the others.

(4) Planck's second law of distribution of energy in the black spectrum,

$$E = c_1 \lambda^{-5} \left(e^{\frac{c_2}{\lambda T}} - 1\right)^{-1} .$$

Before making application of these laws to the problem of estimating inaccessible temperatures, it will contribute much to a clear understanding of the relation of the formulæ which have been examined, to each other, if we leave the chronological development for a moment and briefly consider the general character and limitations of the radiation function—a subject upon which the literature of radiation is singularly barren.[1]

With the help of the theory of functions it is possible to construct expressions which will represent all the necessary con-

[1] An investigation of this character has been made by DR. P. G. NUTTING; see ASTROPHYSICAL JOURNAL, **12**, 208, 1900, and *Phil. Mag.*, (6) **2**, 379, 1901.

ditions of the problem and at the same time permit of a detailed analysis of the component functions of which the general form is made up.

Experiment shows that for any particular temperature (T) there is a corresponding distribution of energy in the spectrum of the black body. If we plot the energy of the radiation (E) for any particular wave-length as ordinate, and the corresponding wave-length (λ) as abscissa, we obtain for any temperature ($T_1, T_2 \ldots T_n$) curves $o\, a_1\, T_1 \infty$, $o\, a_2\, T_2 \infty, \ldots \ldots o\, a_n\, T_n \infty$ of the form indicated in the diagram. All of the curves representing the relation between radiant energy and wave-length are of this general form.

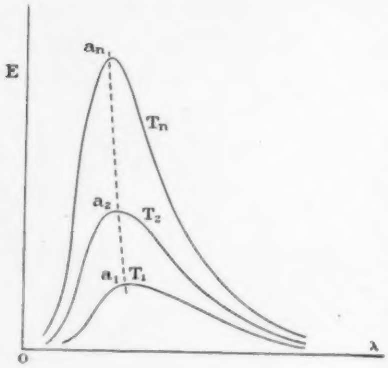


FIG. 4.

Experiment also shows that the locus of the maxima ($a_1, a_2, \ldots \ldots a_n$) in these curves approaches the $+y$ axis with increasing temperature. From the form of these experimental curves, we may say further that the x axis must be tangent to the curve at the points $\lambda = 0$ and $\lambda = +\infty$. Between these points there are no zeros, no negative values, and no infinities—the ordinates are always finite and positive.

In order to define the functions which represent the curves having these properties, it is convenient to define the reciprocal, which evidently also contains no zeros, no negative values, and no infinities except at the points $\lambda = 0$ and $\lambda = +\infty$. We can now apply well-known theorems in the theory of functions and classify the possible expressions according to their singularities. There will be three classes according as the functions contain singularities which are accidental, essential, or a combination of both accidental and essential. They are given in Forsyth's notation in column 2 of the table as shown on the following page.

Character of Singularity	$E^{-1}=\dfrac{1}{f(\lambda)}$ (Forsyth's Notation)	$E=f(\lambda)$	Limiting Conditions
Accidental	(I) $\dfrac{\prod\limits_{r=1}^{m}(\lambda-a_r)}{\prod\limits_{r=1}^{m-p}(\lambda-c_r)}$	(1) $\dfrac{\lambda^q(a_o+a_1\lambda+\ldots\ldots a_r\lambda^r)}{(b_o+b_1\lambda+\ldots\ldots b_s\lambda^s)}$.	$q+r<s$ $q>1$
Essential	(II) $\prod\limits_{i=1}^{n}(\lambda-a_i)^{c_i}\prod\limits_{i=1}^{n}e^{g_1\left(\frac{1}{\lambda-a_i}\right)}$ (III) $G(\lambda)+\sum\limits_{r=1}^{n}G_r\left(\dfrac{1}{\lambda-a_r}\right)$	(2) $cf(\lambda)e^{-g(\lambda)}e^{-g_1\left(\frac{1}{\lambda}\right)}$ (3) $\left[cf(\lambda)e^{g(\lambda)}e^{g_1\left(\frac{1}{\lambda}\right)}+P\left(\lambda,\dfrac{1}{\lambda}\right)\right]^{-1}$	$g(\lambda)>0$ $g_1\left(\dfrac{1}{\lambda}\right)>0$
Accidental and Essential	(IV) $\dfrac{\sum\limits_{i=1}^{n}G_i\left(\dfrac{1}{\lambda-c_i}\right)}{\sum\limits_{i=1}^{n}G_{n+i}\left(\dfrac{1}{\lambda-c_i}\right)}$	(4) $c\lambda^n e^{-g(\lambda)}$ (5) $\left[c\lambda^{-n}e^{g(\lambda)}+P\left(\lambda,\dfrac{1}{\lambda}\right)\right]^{-1}$ (6) $c\lambda^{-n}e^{-g_1\left(\frac{1}{\lambda}\right)}$ (7) $\left[c\lambda^n e^{g_1\left(\frac{1}{\lambda}\right)}+P\left(\lambda,\dfrac{1}{\lambda}\right)\right]^{-1}$ (8) $\ldots\dfrac{P+G}{P_1+G_1}\ldots\ldots\ldots$	$n>1$ $g(\lambda)>0$ $n>0$ $g_1\left(\dfrac{1}{\lambda}\right)>0$ Order of denom. > order of num.

The expressions in column 3 are obtained from the corresponding expressions in column 2 after substituting the boundary conditions. The necessary substitutions and transformations will be considered in detail for each particular class.

(1) *Accidental singularities.*—The first equation is an application[1] of the theorem that a function containing accidental singularities only must have as many zeros as infinities. $\lambda=+\infty$ is a pole of order p. There are m zeros (a_r) and $(m-p)$ poles (c_r) in the finite part of the plane. Since there are no undulations in the curve, the quantities a_r and c_r must be either zero or negative. Expressing the indicated product relations and

[1] FORSYTH'S *Theory of Functions*, 2d ed., Cor. III, p. 81.

taking the reciprocals, we may write the equation in the form (I), in which $q + r < s$.

(2) *Essential singularities.*—Equation III is obtained from II by the addition of polynomial or transcendental terms.[1] Equation II is the most general simple product relation[2] between λ and E^{-1}. For real variables, c_i is any constant, positive or negative, integral or fractional. a_i is the point at which the function becomes discontinuous. When $a_i = +\infty$, theory requires that there be a factor

$$F_1(\lambda) = ce^{g(\lambda)} .$$

And when $a_i = 0$, the factor is

$$F_2(\lambda) = \lambda^{c_i} e^{-g_1\left(\frac{1}{\lambda}\right)} .$$

The reciprocal of the product of these two expressions

$$E = c\lambda^{-c_i} e^{-g(\lambda)} e^{-g_1\left(\frac{1}{\lambda}\right)}$$

is the most general product relation in a single term, which has the required zeros. Since c_i may have any value, we may put

$$f(\lambda) = \lambda^{-c_i}(b_0 + b_1\lambda + \ldots b_n\lambda^n)$$

and then replace the above equation by

$$E = cf(\lambda)e^{-g(\lambda)} e^{-g_1\left(\frac{\lambda}{1}\right)} .$$

$f(\lambda)$ is necessarily polynomial. $g(\lambda)$ and $g_1\left(\frac{1}{\lambda}\right)$ may be either polynomial or transcendental. The roots of the polynomials must occur at the origin or in the negative part of the plane. In general, we know that $g(\lambda)$ must vanish or reduce to a constant when $\lambda = 0$ and become infinite when $\lambda = +\infty$. Similarly $g_1\left(\frac{1}{\lambda}\right)$ must become infinite for $\lambda = 0$, and must vanish or reduce to a constant when $\lambda = +\infty$. $g(\lambda)$ and $g_1\left(\frac{1}{\lambda}\right)$ must therefore always be greater than zero.

(3) *Accidental and essential singularities.*—Equation (IV)[3] is the most general form of functions of this type. It may be composed of equations of the forms (I), (II), and (III), and may, in certain cases, contain only accidental or essential singularities.

[1] *Loc. cit.*, p. 113. [2] *Loc. cit.*, pp. 116–117. [3] *Loc. cit.*, pp. 122–124.

The exponent of λ must be positive in equation (4), and negative in equation (6). The denominator of (8) must be of an order higher than the numerator.

Limiting conditions.—The equations (1–8) include curves which intersect the x axis in addition to those which have the x axis for a tangent at the points $\lambda = 0$ and $\lambda = +\infty$. In order to eliminate the former, we impose the condition that the derivative $\left(\frac{dE}{d\lambda}\right)$ must vanish with the function at these points. The last column contains the required conditions. They are readily determined by inspection.

The condition at the maximum.—We have now obtained all the curves of the required form. It remains to select such only of these curves as have their maximum points on a certain locus which may be expressed as a function of λ and T. Both theory and experiment indicate (Wien's displacement law) that this locus is a function of the product (λT), but it is not necessary to limit the investigation to this particular case.

To impose any condition at the maximum in equations (2–7) it is sufficient to make

$$\frac{\partial}{\partial\lambda}\left[cf(\lambda)\,e^{g(\lambda)}e^{g_1\left(\frac{1}{\lambda}\right)} + P_1(\lambda) + P_2\left(\frac{1}{\lambda}\right)\right] \equiv F(\lambda, T) = 0 \ .$$

The exponents $g(\lambda)$ and $g_1\left(\frac{1}{\lambda}\right)$ must necessarily contain the required function $F(\lambda, T)$, hence

$$(a) \qquad c\frac{\partial}{\partial\lambda}f(\lambda) + cf(\lambda)\left[\frac{\partial}{\partial\lambda}g(\lambda) + \frac{\partial}{\partial\lambda}g_1\left(\frac{1}{\lambda}\right)\right] + \frac{\partial}{\partial\lambda}P_1(\lambda) + \frac{\partial}{\partial\lambda}P_2\left(\frac{1}{\lambda}\right) \equiv F(\lambda, T) = 0 \ .$$

That is, we obtain any required condition at the maximum by putting the corresponding function in the exponents, and then determining the polynomials f, P_1, and P_2 in such manner that equation (a) reduces to a sum of constant terms or to the required function of λ and T.

There are two important special cases, viz.:

$$P_1(\lambda) + P_2\left(\frac{1}{\lambda}\right) = \text{const. or } kf(\lambda) \ .$$

Substituting in (a) we have

$$(b) \qquad \Big[c+k\Big]\frac{\partial}{\partial\lambda}f(\lambda)+cf(\lambda)\frac{\partial}{\partial\lambda}\Big[g(\lambda)+g_1\Big(\frac{1}{\lambda}\Big)\Big]\equiv F(\lambda,\ T)=0\ ,$$

in which $c+k$ is replaced by c for the first case. If $f(\lambda)$ is a monomial, as appears from Stefan's law, equation (b) becomes

$$g(\lambda)+g_1\Big(\frac{1}{\lambda}\Big)\equiv F(\lambda,\ T)=\text{constant}\ .$$

Hence for these special cases the condition at the maximum is determined by the exponents $g(\lambda)$ and $g_1\Big(\frac{1}{\lambda}\Big)$, and is independent of the factor T in the monomial $f(\lambda)$.

Suppose we now put

$$\lambda^m T^n=\text{constant.}$$

$$f(\lambda)=\lambda^p T^q=\frac{1}{k}\Big[P_1(\lambda)+P_2\Big(\frac{1}{\lambda}\Big)\Big]$$

then (3) may be written,

$$(c) \qquad E=\Big[c_0\lambda^p T^q\Big(e^{g(\lambda^m T^n)}e^{g_1\left(\frac{1}{\lambda^m T^n}\right)}+k\Big)\Big]^{-1}.$$

And to obtain Planck's last equation, for example, we have only to insert those special relations which appear from the foregoing to be established:

$p=5$ (Stefan-Boltzmann law)
$q=g(\lambda^m T^n)=0,\ k=-1$
$m=n=1$ (Wien's displacement law)

when

$$E=\Big[c_1\lambda^{-5}(e^{\frac{c_2}{\lambda T}}-1)\Big]^{-1} \qquad \text{[Planck]}$$

All the other spectral distribution equations are obtainable by inspection in the same way by introducing the characteristic limitation of each in the following:

$$g(\lambda^m T^n)=k=0,\quad f(\lambda)=\lambda^p T^q,\quad \lambda^m T^n=\text{constant.},$$

then

$$E = c_1 T^{\frac{3}{2}} \lambda^{-6} e^{-\frac{c_2}{\lambda^2 T}} \qquad \text{[Michelson]}$$

$$E = c_1 \lambda^{-2} e^{aT - \frac{1}{b^2 \lambda^2 T^2}} \qquad \text{[Weber]}$$

$$E = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}} \qquad \text{[Wien]}$$

$$E = c_1 (\lambda T)^{\frac{1}{2}} \lambda^{-5} e^{-\frac{c_2}{\lambda T}} \qquad \text{[Thiesen]}$$

$$E = c_1 T \lambda^{-4} e^{-\frac{c_2}{\lambda T}} \qquad \text{[Rayleigh]}$$

and

$$E = c_1 \lambda^{-4} T e^{-\frac{c}{(\lambda T)^v}} \quad 1.2 < v < 1.3 \quad \text{[Jahnke]}$$

Köveslighety's equation[1]

$$E = \frac{A \lambda^2 T^4}{(\lambda^2 T^2 + c^2)^2}$$

is a special form of (1).

With but one exception, all of the equations that have been deduced to represent the relations between radiated energy and wave-length, are seen to be very special cases of (7) or (c), in which $g(\lambda) = 0$. If the number of constants is limited to two in this way, by placing $g(\lambda)$ or $g_1\left(\frac{1}{\lambda}\right)$ equal to zero, the equation (c) is the most general relation possible of this type. Greater generality might be obtained by means of equation (8) or by introducing other conditions at the maximum in (7). Planck's equation therefore represents, in this direction, the greatest generality obtainable with but two physical constants, the Stefen-Boltzmann and Wien (displacement) relations.

It would be perfectly possible to define the emission function completely, provided the boundary conditions could be physically defined. The equations which have been deduced have a zero of infinite multiplicity at the origin, and a zero of finite multiplicity at $\lambda = +\infty$. The latter becomes finite when $g(\lambda)$ is put equal to zero in (3). If this can be shown to be a neces-

[1] R. v. KÖVESLIGHETY, *Grundzüge einer theoretischen Spectralanalyse*, Halle, 1890.

sary condition, and if a definite physical meaning can be assigned to these zeros, the required function is completely determined by means of function theory alone in equation (7). Equation (5) is the complementary equation with a zero of finite multiplicity at the origin and of infinite multiplicity at infinity, while equation (3), just discussed, represents the general case with both zeros of infinite order.

The question now arises whether the temperature of the Sun or of inaccessible volcanic openings can in any proper sense be measured, or even effectively estimated, with the help of the laws we have been considering. It is not usual to apply the same reasoning in attempting to answer this question, which would be used in studying the temperature of the incandescent lamp, for example. The Sun is so completely inaccessible to the common methods of pyrometry, and, moreover, its temperature has so important a rôle in the fundamental conclusions of geology, meteorology, and astronomy, that a new method of estimating it, even if only indirectly or imperfectly applicable, is accustomed to be pressed into service with little question and the exceptional conditions put forward in benevolent extenuation of the outcome if necessary. This has been done with the laws of black radiation.[1] There are many papers, and very recently even text-books, which apply the equations of the black body directly to the measurement of the solar temperature, with no more justification than the fact that we have well-founded and experimentally verified laws of radiation, and therefore have but to apply probable corrections for the absorption which the solar radiation may have suffered before reaching us, to obtain the probable solar temperature.

Against this simple interpretation of the problem there are two serious objections which we certainly are not justified in overlooking, whatever attitude we subsequently choose to assume toward them.

(1) The radiation from the Sun is certainly not pure temperature radiation, but partakes of the character of luminescence,

[1] Some computations of the solar temperature based upon recent determinations of the constants in the equations of the black body are appended at the end of the paper.

between which and the black body no temperature relation has ever been found.

(2) The temperature of the Sun is certainly so high as to require at least a fivefold extrapolation of the temperature scale upon which the estimate is usually based—an operation which, for a function like temperature, it is difficult to interpret theoretically and which can hardly be called measurement practically.

As long ago as 1847, when Draper[1] published his conclusion that solid bodies begin to emit luminous radiation of the same wave-length at the same temperature, he excepted lime, marble, fluorspar, and such substances as emit radiant energy in no apparent relation to the temperature. Radiation of this character is familiar enough from the glow of the fire-fly and the phosphorescence of decaying vegetable matter, to the radiation sent out from more or less rarified gases during electrical discharges through tubes, and the large class of phenomena in which the source of the radiant energy is not heat, but chemical or photo-chemical change. In all these, radiant energy is emitted and varies widely both in wave-length and intensity with little or no corresponding change in the temperature.

Nor is the bounding line between these two great classes of radiation in the least indistinct or indefinite. All bodies emitting purely calorific radiation, *i. e.*, radiation in which the energy is supplied by heat alone without infusion of electrical or chemical energy, follow Kirchhoff's law that the ratio of radiating and absorbing powers for a given wave-length and temperature is the same, and equal to the radiating power of the black body. Solids and liquids which can be heated to incandescence without complicating phenomena will generally be of this class. All bodies in which the energy radiated is supplied in whole or in part from other sources—chemical or electrical activity, for example—do not follow Kirchhoff's law. This kind of radiant energy is generally termed "luminescence" and bears no simple relation to the temperature. The radiation from gases is usually wholly or partly luminescent. E. Pringsheim,[2] in his paper

[1] *Loc. cit.* [2] *Rapports congrès international*, **2**, 100 *seq.*, 1900.

"L'émission des gaz," for the international congress at Paris (1900), distinguishes three kinds of radiation from gases:

(1) The phenomena which produce emission merely serve to bring the gas to the high temperature necessary for such emission. To this class the second law of thermodynamics is applicable; hence also Kirchhoff's law and the entire system of reasoning which has been reviewed.

(2) The electrical or chemical phenomena which produce the emission bring the gas into a somewhat modified state in which its radiating and absorbing powers differ with different initiating phenomena. Here Kirchhoff's law might be fulfilled in special cases.

(3) The electrical and chemical phenomena furnish entirely or partially the energy for the emission. Here the law of Kirchhoff finds no application.

If we now apply this analysis to solar radiation, and bear in mind that the surface layers in which the radiant energy for a temperature determination must originate are certainly gaseous, we see that the study of black radiation offers no royal road to the determination of the temperature of the Sun. It may yield a valuable approximation, but it is difficult even to determine just how valuable. Scheiner, almost alone among writers in this field, frankly acknowledges that our established generalizations upon black radiation are not properly applied to the determination of the temperature of the Sun, although he gives them in full and computes the solar temperature from them in various ways. He says:[1]

> On account of our lack of knowledge of the emitting power of the photosphere, a conclusion regarding its true temperature is not possible, and even if the emitting power were known, the real temperature would remain doubtful, since Stefan's law has been proved only for radiation from an absolutely black body.

The same analysis applies to the determination of terrestrial volcanic temperatures. We do not know the radiating power of the molten masses which now and then become exposed in volcanic openings, but the phenomena are or soon will be quite

[1] See *Strahlung und Temperatur der Sonne*, Leipzig, 1899, p. 50.

within the reach of laboratory research. In most cases the radiation is undoubtedly almost entirely calorific, and where this can be assumed with reasonable certainty, temperature estimates become correspondingly reliable, for the conditions of blackness are often remarkably well fulfilled in volcanic phenomena.

Let us now turn to the second difficulty in establishing a definite solar temperature. It is apparently a simple matter, but it is worth while asking[1] just what it means to say that the temperature of the Sun is 6000° or 8000° or 10000°. Temperature is not an additive function. We cannot take a number of bodies of the same temperature and by placing them in juxtaposition obtain a different temperature by summation as we lay a dozen foot-measures across a room and say the room is twelve feet long. We can obtain a measure of the temperature only through some phenomenon which varies with the temperature, and even then we must assure ourselves in some way that the phenomenon varies *uniformly* with equal increments of heat before we can make effective use of it. In the early researches the expansion of mercury was in most common use for this purpose, but it is of limited application and is not quite uniform in its expansion. Hydrogen was substituted after Lord Kelvin had shown that the expansion of a perfect gas would furnish an ideal temperature scale, but hydrogen is not an absolutely perfect gas. The international scale of temperature, as it is called, is now, by general agreement, based upon the expansion of hydrogen between the melting-point of ice and the boiling-point of water, divided into one hundred equal increments or degrees.

The measurement of the expansion of hydrogen has been continued up to six hundred such degrees,[2] but beyond this point no satisfactory containing vessel has been found for it. A less active and also less perfect gas — nitrogen — has been substituted, which has enabled the expansion measurements to be continued

[1] See also E. BUCKINGHAM, *Monthly Weather Review*, April 1903.

[2] P. CHAPPINS et J. A. HARKER, "Comparaison du thermomètre à résistance de platine avec le thermomètre à gaz, et détermination du point d'ébullition du soufre." *Travaux et mémoires du bureau international des poids et mesures*, **12**, 1–89; also *Phil. Trans.*, **194**, 37, 1900.

to 1150°.[1] The extension of the measurements upon hydrogen to 600° is proper extrapolation, and inasmuch as all gases approach the perfect gas at the higher temperatures, the continuation to 1150° with nitrogen may also be so regarded, but here trustworthy gas measurements end, and the extrapolated temperature scale on the basis of international definition ends also. The probability of its being continued upward for any considerable interval is small.

In choosing some other phenomenon which will vary with sensible uniformity as the temperature increases above this point, there is no limitation upon the selection except the convenience of the observer and the needs of his problem. To facilitate comparisons of results among widely separated scientists, it is desirable to retain the size of the unit or degree as nearly as possible, and one of the most common schemes is this: take a thermo-couple or element of suitable refractory metals (platinum with platin-rhodium), inclose it in a furnace[2] with the bulb of the gas thermometer used above, and record the electromotive force at short intervals as far as the gas expansion can be measured. Then, *assuming* that the thermo-electric electromotive force will continue to increase in the ratio thus obtained, five hundred or more "degrees" above the highest point of the gas scale can be conveniently established. There is good reason to believe that these thermo-electric degrees agree well with the degrees of the gas scale, though in principle they form an independent system, merely numbered to conform to the generally adopted standard for the lower temperatures. This system of measurement also admits of but little extension beyond the present limit, unless more refractory thermo-electric materials can be found to replace the present ones.

Beyond this point (about 1600°) there is probably no better temperature phenomenon available for the purpose than the radiation from the black body which has just been considered. The comparison with the gas and thermo-electric scales over the long range from 0° to 1550° we have seen to be very satisfactory. And if the theoretical arguments which have been adduced in their sup-

[1] HOLBORN and DAY, *loc. cit.*, 1900. [2] HOLBORN and DAY, *loc. cit.*

port can be considered sufficiently conclusive, we have a radiation scale which will suffice for the highest temperature at which a black body can exist as such. So far as the authors are aware, but a single attempt has been made to do this, however, and only one temperature was established. Messrs. Lummer and Pringsheim[1] prepared a thin, hollow tube of carbon, surrounded it with an atmosphere of nitrogen, heated it electrically to a temperature corresponding to about 2000° C., and made three series of measurements of its temperature based upon three different formulæ for black radiation. The apparatus held out only through one short heating, but the three sets of results when reduced showed a maximum difference of only 1 per cent. (20°), although the temperature was almost twice as high as the nearest point on the gas scale (1150°). It was a splendid effort, and the excellent agreement of the three extrapolations among themselves goes far to establish the Stefan-Boltzmann law and the Wien displacement theorem as entirely general relations. A table of Lummer and Pringsheim's temperatures follows:

Method	T Absolute
Photometric	2310
Total radiation	2325
Photometric	2320
Total radiation	2330
Energy maximum	2330
Photometric	2330
Total radiation	2345
Energy maximum	2320

But if we take the Stefan-Boltzmann law, which undoubtedly rests upon the simplest and most secure fundamental reasoning of all the existing radiation equations, express its temperature relation in terms of the extended gas scale which reaches 1150°, assume it to be faultlessly supported by the thermo-electric extension to 1550° and by two other radiational formulæ in a single measurement at 2000°, and even assuming, further, that the Sun were an unquestioned black body, are we justified in saying that

[1] O. Lummer and E. Pringsheim, "Die strahlungstheoretische Temperaturskala und ihre Verwirklichung bis 2300° abs.," *Vehr. Deutsch. Phys. Ges.*, (5) **1**, 3, 1903.

the temperature of the Sun, as determined from its radiation, is 6000° or 8000° or 10000° *upon the gas scale?* To extrapolate any well-established functional relation eight or ten times as far as it is supported by physical fact would require to be justified by very exceptional reasons or conditions, but to extrapolate one temperature phenomenon by a totally different one, so far into a domain inaccessible to all corroborative physical measurement, is quite meaningless. We have no temperature of 10000° in the sense in which we have a temperature of 1000°. If we are prepared, upon the basis of existing evidence, to consider the radiation scale established as a natural law, as has been proposed,[1] we might of course define all temperature in terms of it, including that of the Sun, if it could be shown to be a black body. These laws of radiant energy have unquestionably given us the best existing high-temperature scale—a scale which, in one most important particular, is a distinct advance over all other scales now in use: it is independent of the properties of any particular substance—but they are strictly confined to calorific radiation and are not properly used for the indefinite extrapolation of the gas scale.

There is another difficulty in the application of the laws of the black body to temperature measurement which is of a very practical character—calorific radiation is not usually black. In particular cases (lamp-black, platinum black) it approaches blackness, but all known substances possess limited properties of absorption and radiation, and cannot be reached directly until the law of distribution of energy in the spectrum of the black body is finally established. The error which arises in applying the Stefan-Boltzmann law to calorific radiation which is not black is best shown by citing the extreme case. Of all known refractory substances emitting pure temperature radiation, incandescent platinum reflects the largest percentage of the incident energy, and is therefore farthest removed from absolute blackness. Lummer and Pringsheim[2] have made a number of measurements

[1] O. Lummer and E. Pringsheim, *Vehr. d. Deutsch. Phys. Ges.*, (5) **1**, 3–13, 1903.

[2] "Die Verteilung der Energie im Spektrum des schwarzen Körpers u. des blanken Platins," *Verh. d. Deutsch. Phys. Ges.*, **1**, 215–230, 1899.

upon platinum, a set of which ($T_{min.}$) is compared with similar measurements upon the black body ($T_{max.}$) in the table below:

	λ_m	$T_{max.}$	$T_{min.}$
Arc light	0.7 μ	4200° abs.	3750° abs.
Nernst lamp	1.2	2450	2200
Welsbach mantle	1.2	2450	2200
Incandescent lamp	1.4	2100	1875
Candle	1.5	1960	1750
Argand gas burner	1.55	1900	1700

From these it appears that the maximum error can amount to 225° at 2000°, or about 10 per cent.—a quantity of inconsiderable magnitude compared with the differences in the earlier measurements of these quantities, and one which admits of correction in all particular cases. If a black body can be raised to the required temperature, or if the temperature be contained in an inclosed space which fairly fulfils Kirchhoff's conditions for blackness, excellent measurements of extremely high temperatures are possible. A very simple device for measuring much higher temperatures than have heretofore been accessible, and this without any form of contact between heated body and measuring apparatus, is due to Holborn and Kurlbaum,[1] and may be briefly described. A cheap telescope may be mounted with a tiny incandescent lamp in the focus of the object-glass and a monochromatic, transparent screen before the eyepiece. The lamp is fed with a measured current through an amperemeter and rheostat. Direct the telescope at a black body of known temperature (measured by bolometer or thermo-element) and vary the current in the lamp until it disappears against the black (perfectly absorbing, not necessarily black in color) background, and note the current. The disappearance is sharp and complete. Repeat the measurement for several temperatures and tabulate or plot them in terms of the current intensities. The temperatures of other bodies emitting calorific radiation can then be found by merely directing the telescope at them, regulating the current until the carbon filament disappears against the background, and

[1] *Ann. d. Phys.*, **10**, 225–241, 1903.

noting the temperature which corresponds to the indicated current. One or two approximations are involved, and considerable refinements will suggest themselves in practice, but the remarkable simplicity of the apparatus, and the convenience with which completely inaccessible temperatures are measured, will serve to illustrate the possibilities of the radiation scale for an almost unlimited range of temperature measurement.

Warburg,[1] in a recent paper before the Deutsche Physikalische Gesellschaft of Berlin, computes the solar temperature as follows: He assumes (1) that the rate of radiation per degree is constant, and (2) that the Stefan-Boltzmann law is applicable at all temperatures.

Let H_t = radiant energy corresponding to temperature t,
ϕ = apparent diameter of Sun $= 0^\circ\ 32'$,
s = solar constant,
h_1 = rate of radiation,
$= 0.0176\ \frac{\text{gr. cal.}}{\text{cm}^2\text{sec.}}$ (Kurlbaum).

As a consequence of assumption (1), we have

$$\frac{s}{h_1} = \left[\frac{H_t - H_o}{H_{100} - H_o}\right] \sin^2 \frac{\phi}{2}\ , \tag{1}$$

and from the Stefan-Boltzmann law,

$$\frac{H_t}{H_o} = \left[\frac{273 + t}{273}\right]^4 , \tag{2}$$

which substituted in (1) gives approximately

$$t = 273 \sqrt[4]{\frac{2.483}{\sin^2 \frac{\phi}{2}} \cdot \frac{s}{h^1}} - 273\ .$$

From which, using the value of the solar constant obtained by

[1] *Verh. Deutch. Phys. Ges.*, **1**, **2**, 50, 1899. See also SCHEINER, *Strahlung und Temperatur der Sonne*, Engelmann, Leipzig, 1899; and F. H. BIGELOW, "Eclipse Meteorology and Allied Problems," *Bull. I*, U. S. Dept. Agriculture, 1902.

Langley in his Mount Whitney observations[1] (14,898 feet elevation), 3.07 calories per sq. cm. per minute, $s = \frac{3.07}{60}$, and

$$t = 6286^\circ \text{ C.}$$

Using his more recent tentative value[2] (which, by the way, he describes as probably somewhat too low), based upon observations near the sea level, $s = \frac{2.54}{60}$,
and

$$t = 5983^\circ \text{ C.}$$

A series of very consistent determinations just published by Abbot[3] give a still lower value for the solar constant, $2.17 \frac{\text{cal.}}{\text{cm}^2}$.
Substituting this value, $s = \frac{2.17}{60}$
and

$$t = 5741^\circ \text{ C.}$$[4]

The second estimate is very simply made from the Wien law of the energy maximum:

$$\lambda_m T = 2940 \text{ (Lummer) and } \lambda_m = 0.49\mu \text{ (Abbot).}$$

Substituting these,

$$T = 6000^\circ \text{ (abs.)} .$$

Paschen's[5] numerous determinations of this constant give 2921 as the mean value. Substituting

$$T = 5961^\circ \text{ (abs.)} .$$

These assume that the Sun is a perfectly black body. Now the refractory substance farthest removed from the black body in its radiating properties, that is, the substance which reflects

[1] S. P. LANGLEY, "Researches on Solar Heat," Professional Papers of the Signal Service, No. 15.

[2] S. P. LANGLEY, "The Solar Constant and Related Problems," ASTROPHYSICAL JOURNAL, **17**, 89–99, 1903.

[3] C. G. ABBOT, *Smithsonian Miscellaneous Collections*, **45**, 74, 1903 (December 9).

[4] The Ångström value of the solar constant, 4 cal. per sq. cm, although probably more extensively quoted than any other, is undoubtedly much too high. (See ABBOT, *loc. cit.*).

[5] F. PASCHEN, "Ueber das Strahlungsgesetz des schwarzen Körpers," *Ann. d. Phys.* (4), **1**, 277, 1901.

the largest percentage of radiant energy which falls upon it, is bright platinum. If the Sun is not a black body, but still is to be classed among purely calorific radiators, it must lie between the black body and platinum in its radiating properties. Considering the Sun to have only the radiating power of platinum, we should have, assuming Lummer's constant,

$$\lambda_m T = 2630 \text{ for platinum, and } \lambda_m = 0.49\mu \ ,$$
$$T = 5367^\circ \text{ (abs.)} \ ,$$

which may fairly be called the lower limit of possibility under the laws of radiant energy.

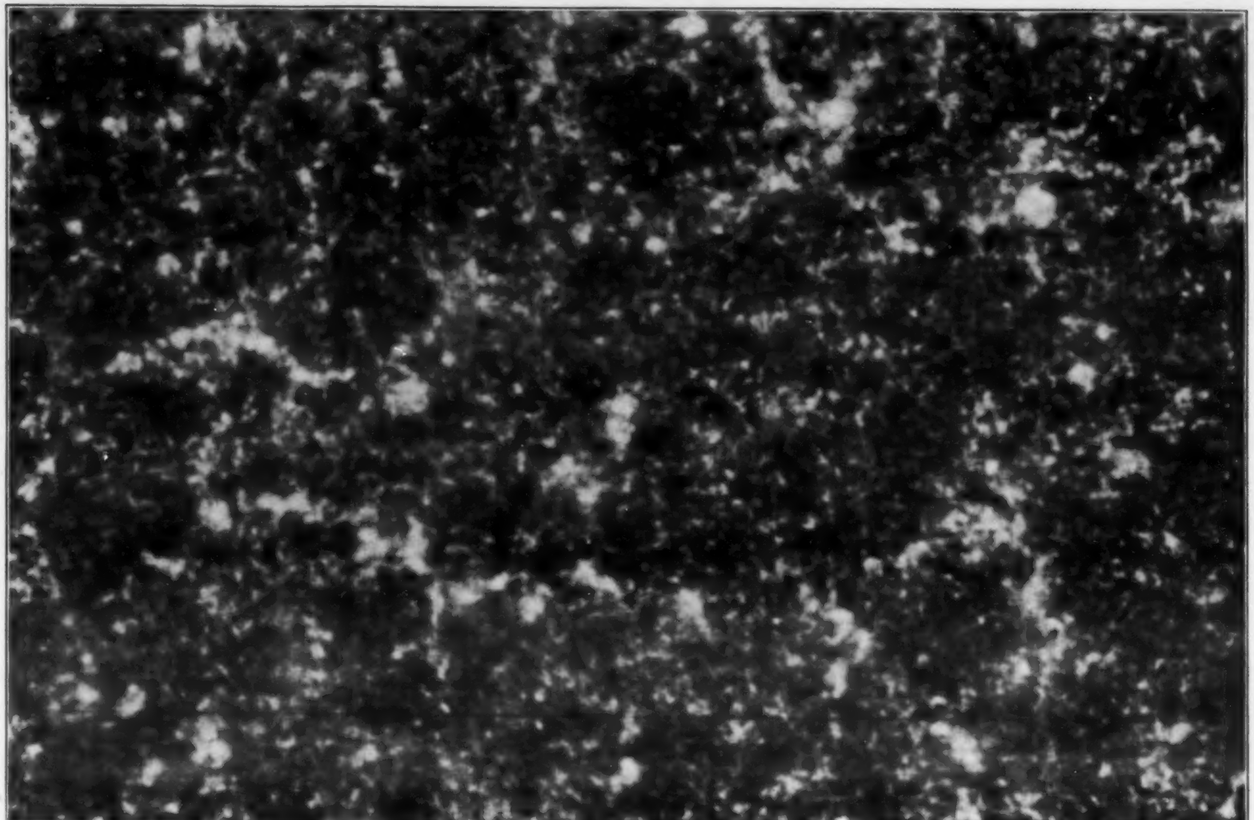
The last three temperatures are in terms of the absolute scale, and require to be diminished by 273° for expression in centigrade degrees. Taking all these values together, the laws of black radiation would appear to place the Sun at a temperature somewhat below 6000° C.

It is hardly fair to subject estimates of this character to such criticism as is usually applied to physical measurements, but it is plain that, assuming the validity of the established generalizations upon black bodies, they are applicable to the Sun, and that such extravagant extrapolation is justified by the inaccessible character of the Sun for such study, the differences in the values of the solar constant are still much too large.

It is noteworthy that as the limits of physical measurement have been extended, the estimates of the solar temperature have been gradually diminishing from 100000° and higher, until now we have a temperature almost within reach of terrestrial resources. The temperature of the electric arc is about 4000°, measured by the same methods.

U. S. GEOLOGICAL SURVEY,
Washington, D. C.,
November 1903.

PLATE II.



Minute Structure of the Calcium Flocculi at H_2 Level. 1903, September 22.

(Scale: Sun's Diameter = 0.890 Meter.)

CALCIUM AND HYDROGEN FLOCCULI.

By George E. Hale and Ferdinand Ellerman.

In the Astrophysical Journal for November 1902 a brief account was given of the solar work then in progress at the Yerkes Observatory.[1] The paper contains a photograph of the Rumford spectroheliograph, but gives no detailed description of this instrument. Such a description has now been published as Part I, Volume III, of the *Publications of the Yerkes Observatory*. To this we may refer for details regarding the instrument and results; we propose to give here only a brief description of some of the results recently obtained, accompanied by reproductions of photographs from the paper in Volume III.

On account of the fact that the diameter of the solar image in the focal plane of the forty-inch refractor is more than three times as great as that of the image given by the twelve-inch Kenwood telescope, and also because of improvements in both the optical and mechanical construction of the Rumford spectroheliograph, the present results are much superior to those formerly obtained at the Kenwood Observatory. The Kenwood photographs show not only the larger regions of calcium vapor on the Sun's surface, but also those smaller elements that constitute a kind of reticulation, which covers the Sun from pole to pole. But the scale of the earlier photographs was quite insufficient to show the minute structure, resembling the granulation of the photosphere, which is clearly visible on all photographs taken under good atmospheric conditions with the Rumford spectroheliograph. Plate II, taken with the slit set at the center of the K line, shows this granulated appearance very clearly. It is also well shown in Fig. 2, Plate IV, where the size of the minute flocculi may be determined by the aid of the scale; but it does not appear to advantage in Plate I, on account of the comparatively large size of the squares in the half-tone

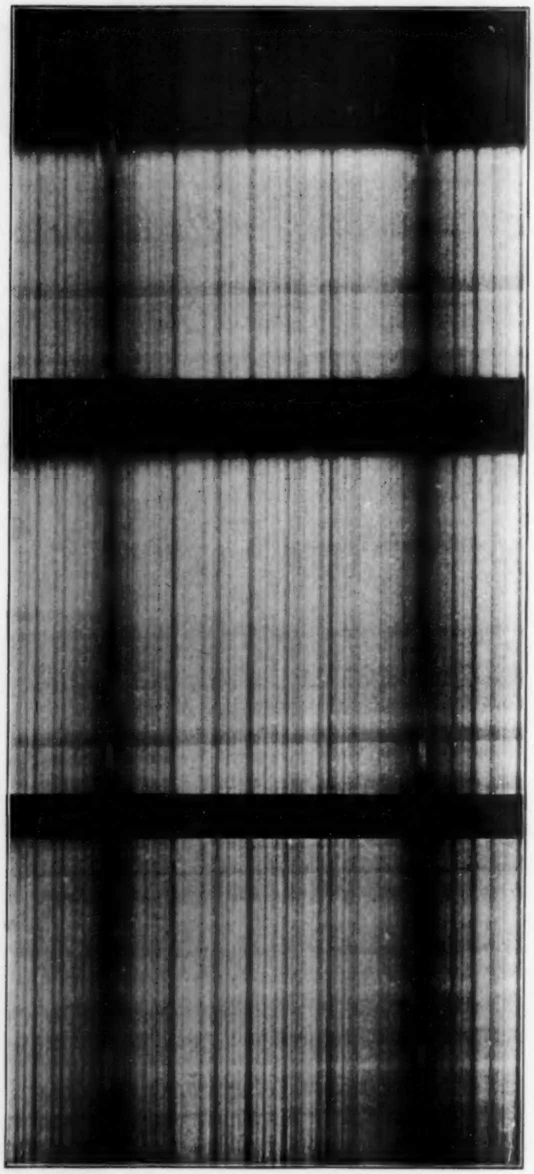
[1] George E. Hale, "Solar Research at the Yerkes Observatory," Astrophysical Journal, November, 1902.

screen. The question at once arises: What relation does this granulated structure bear to that of the photosphere? More specifically, are the small luminous masses of calcium vapor to be regarded as roughly spherical in form, in accordance with Janssen's view of the structure of the grains of the photosphere, or are they to be considered as the summits of columns of calcium vapor, rising from the upper portion of those columns of condensed vapors which, in Langley's view, produce the well-known photospheric granulation?

At this point it should be remarked that the name "faculæ," formerly employed by one of us to designate the calcium clouds photographed with the spectroheliograph, is no longer to be regarded as suitable. Indeed, this fact was long ago pointed out by Deslandres. But in the earlier stages of the work it did not seem desirable to introduce a new term, especially as the calcium regions are closely related with the faculæ, and frequently are exactly similar to them in form. Nevertheless, the calcium vapor, as is clearly shown by the results given in this paper, occupies a region higher above the photosphere than that occupied by the faculæ. It has now seemed best, in order to avoid any possible confusion in the future, to suggest the name *flocculus* to distinguish a mass of vapor from the facula which may or may not lie below it. The *faculæ*, then, are elevated regions of the photosphere, which may be seen with the eye and on direct photographs when not too near the center of the Sun, and which are characterized by a continuous spectrum. The *flocculi* are clouds of gas or vapor lying at a higher level, which are not visible in direct observations or in ordinary photographs, but are shown on photographs taken with the spectroheliograph. The term "flocculi" is applied indiscriminately to all bright or dark clouds of vapor photographed in projection on the Sun's disk, without distinction of level. In other words, a flocculus may be a mass of vapor in the reversing layer, or in the chromosphere, or in a prominence. As will be shown in this paper, the forms of the clouds of calcium, hydrogen and other vapors photographed in projection on the Sun's disk are not always the same. For this reason we shall speak of calcium flocculi, hydro-

PLATE III.

H K



(a)

(b)

(c)

H and K Lines on the Disk, in the Chromosphere, and in a Prominence (a).

PLATE IV.

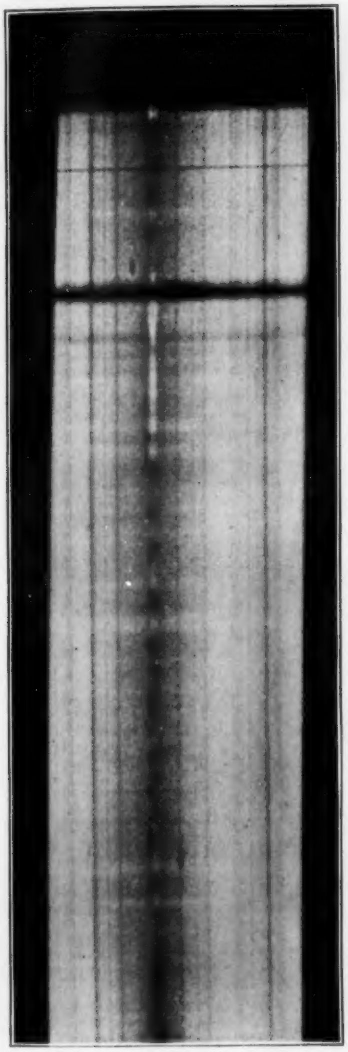


FIG. 1.—K Line on the Disk and at the Limb.

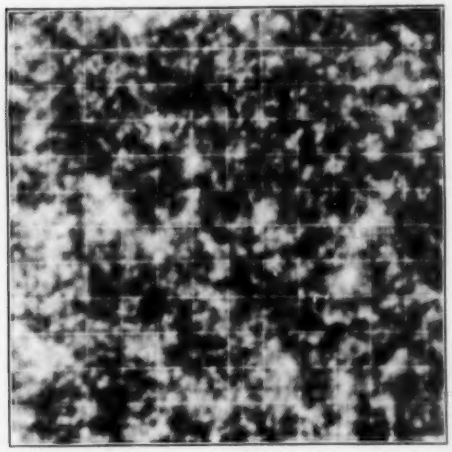


FIG. 2.—Minute Calcium Flocculi (H_2). The Squares are 10" on a Side.

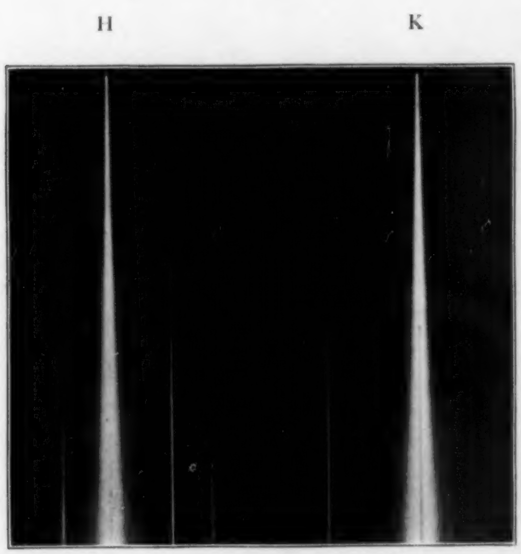


FIG. 3.—H and K Lines in Electric Arc Showing Reversals.

gen flocculi, etc. The method of determining the level represented by a given photograph is described below.

FORM OF THE CALCIUM FLOCCULI AT DIFFERENT LEVELS ABOVE THE PHOTOSPHERE.

In order to determine the nature of the calcium flocculi, it is evident that great advantage would result from the use of a method which would permit sections of the columns of calcium vapor, if such exist, to be photographed at different heights above the photosphere. It is fortunately possible to accomplish this result by the use of a method which appears to have received no previous practical application in the solution of this question, though it was described in an article published in the *Comptes Rendus* by Deslandres in 1894. Deslandres points out in this paper that photographs taken with the second slit of a spectroheliograph set on the broad dark shade of the K line ought to give results different from those obtained when the slit is set on the bright line at the center of K. But he does not refer to the fact that different results will be obtained when the second slit is set at different points on the broad dark shade.

In order to make the matter clear, let us consider Fig. 3, Plate IV, which is reproduced from a photograph of the H and K lines in the spectrum of the electric arc. At the center of the arc, corresponding to the lower part of the photograph, the calcium vapor is dense and the lines are very broad. In the outer part of the arc, corresponding to the upper part of the photograph, the calcium vapor is comparatively rare, and the lines are narrow. The breadth of the lines in the intermediate regions varies with the density of the calcium vapor. The fine dark line of reversal through the center of the lines is produced by the cool, rare calcium vapor in the extreme outer part of the arc.

A similar condition of things exists in the Sun. As will be seen by reference to Plate III and to Fig. 1, Plate IV, we have at H and K in the spectrum of the Sun's disk a composite structure, composed of three parts: (1) a broad, dark band, which we shall designate as H_1 or K_1; (2) a comparatively narrow bright line,

lying at the center of this band, at points on the Sun's disk where the slit crosses hot masses of calcium vapor (H_2, K_2); and (3) a very narrow dark line running through the center of H_2 and K_2 (H_3, K_3). The diffuse dark bands H_1, K_1 are due to the comparatively dense calcium vapor which lies near the photosphere. In general, as we know from eclipse results and from our own photographs of the spectrum of the chromosphere taken in full sunlight with a tangential slit, the dense vapor corresponding to H_1, K_1 lies so low in the chromosphere that it cannot be observed projecting above the Sun's limb. In photographs of the spectrum of the chromosphere taken with radial slit, H_2 and K_2 are seen to project for some distance above the Sun's limb, though they do not extend to the highest part of the chromosphere. In this region, where the calcium vapor is less dense, the lines narrow down so as to give the well-known "arrow-head" appearance; in the upper chromosphere and in prominences they are very narrow, and since the vapor is cooler at these elevations, a narrow dark absorption line (H_3, K_3) is produced at the center of H_2 and K_2.

We are not prepared to explain why H_2 and K_2 appear as bright lines, acting as though the intermediate mass of vapor to which they correspond were hotter than the vapor above and below it. It may be that some electrical or chemical effect is responsible for this appearance, and it would therefore be unsafe to assume that an abnormally high temperature prevails in this intermediate layer.

It is evident that if the second slit of the spectroheliograph is made to correspond with the extreme outer edge of H_1 or K_1, it can receive light only from that calcium vapor which is dense enough to produce a line of this breadth. The rarer vapors, at higher levels, produce narrower lines, which therefore send no light through the second slit. When the slit is set nearer the center of the line it receives light from the vapor at higher levels, and also, of course, from the vapor at lower levels. But as the intensity of radiation of the vapor increases with increasing height above the photosphere, up to the H_2, K_2 level, the successive photographs will in general tend to show sections of the

PLATE V.

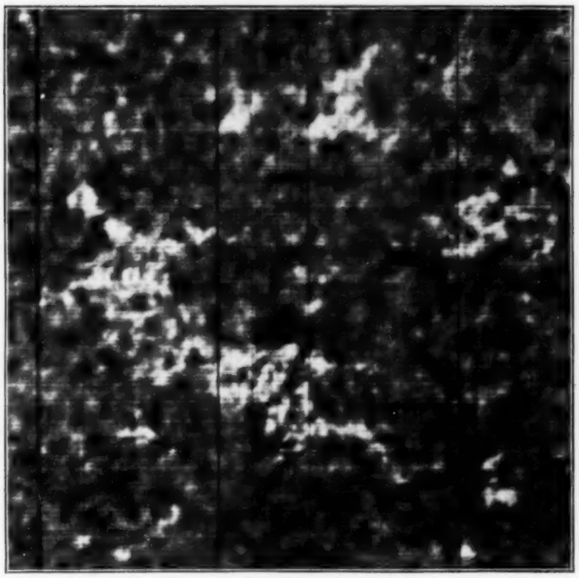


FIG. 1.—3^h 40^m. Low H_1 Level. Slit at $\lambda 3962$.

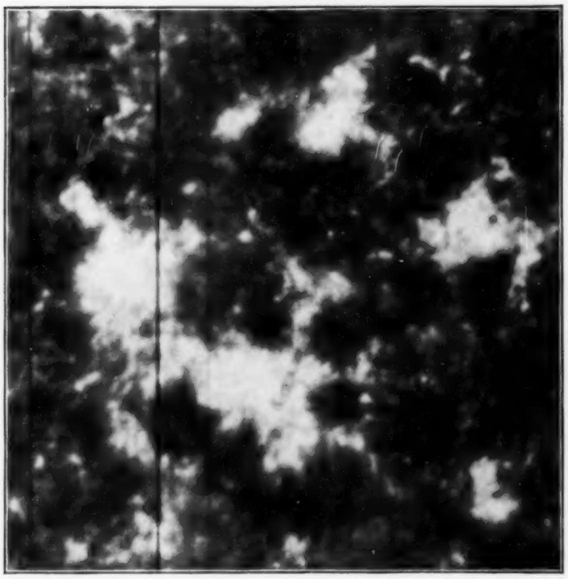


FIG. 2.—3^h 31^m. H_2 Level. Slit at $\lambda 3968.6$. Same Region as Fig. 1.

MINUTE STRUCTURE OF THE CALCIUM FLOCCULI, 1903, SEPTEMBER 22.
(Scale: Sun's Diameter = 0.890 Meter.)

calcium clouds, corresponding to the highest level at which the vapor is dense enough to produce a line sufficiently wide to enter the second slit. It is evident that the cooler vapor of the H_3, K_3 level would not in general be shown in photographs taken with a second slit wide enough to include the whole of the H_2, K_2 line. But in some cases, because of greatly increased width and intensity of H_3, K_3, and the faintness or absence of H_2, K_2, the absorbing vapors of the highest level should be shown.

An examination of the photographs reproduced in Figs. 1 and 2, Plate V, will show how this method has been used to determine the structure of the calcium flocculi. It will be seen that the low-level photograph (Fig. 1) shows a series of comparatively small and well-defined elements at the base of the flocculi, which at the higher level of Fig. 2 expand, and probably overhang, so as to cover a much larger area. From this and other results, obtained under the very perfect atmospheric conditions essential in a case of this kind, it seems probable that the calcium flocculi are in general made up of a series of columns, which expand as they reach higher levels, and in many cases overhang laterally. Before this question can be finally solved, however, it will be necessary to secure a large number of photographs under very perfect atmospheric conditions.

Plates VI and VII will serve to illustrate the method of photographing sections corresponding to different levels. The two photographs of the great Sun-spot of October 1903, which are especially arranged in Plate VIII for purposes of comparison, will serve to give a still better illustration of the method. The high-level photographs in Plates VII and VIII seem to give definite indications of the presence of dark regions, not shown in the low-level photographs, which are not improbably caused by the absorption of cooler vapors at higher levels.

HYDROGEN FLOCCULI.

In employing the spectroheliograph to photograph the Sun's disk through dark lines other than H and K, it is obvious that the principal condition to be fulfilled is that the linear dispersion shall be sufficient to cause the line in question to be wider

than the second slit. If this is not done, it is evident that light from the continuous spectrum on either side of the dark line will reach the photographic plate, and that very little of such light would be sufficient completely to annul the effect produced by variations in intensity of the dark line. For while the dark lines are, of course, dark only by contrast, their light is much less intense than that of the continuous spectrum on which they lie.

A most valuable property of the spectroheliograph, which has not hitherto received special attention, is its power of recording bright or dark regions whose intensity differs so little from that of the general solar surface that the existence of such regions would never be suspected from photographs of spectra alone. The sensitiveness of the instrument in this regard seems to be due to the well-known physiological fact that although differences in intensity of the various parts of a narrow line might not be perceptible, they would become so if the line were greatly widened. Through this property of the spectroheliograph, it becomes possible to photograph hydrogen and metallic flocculi not previously recognized with the spectroscope or by other methods of observation.

In May of the present year the Rumford spectroheliograph was so modified as to render it available for photography with some of the wider dark lines, notably those of hydrogen. The optical train of the instrument, as ordinarily used with the H and K lines, consists of a plane mirror, which reflects the parallel rays coming from the collimator lens to the first of two 60° prisms of light flint glass (Fig. 1). The total deviation of the ray employed, after refraction by the prisms, is 180°. For work with the dark lines we decided, after some experimenting, to substitute a plane grating ruled with 20,000 lines to the inch for the mirror of the above combination. The position of the grating is such as to cause any desired region in the first-order spectrum to fall on the first prism, after which it is further dispersed by the two prisms of the train. The advantage of this combination is not only to give greatly increased dispersion, but also to reduce very materially the diffuse light of the grating,

PLATE VI.

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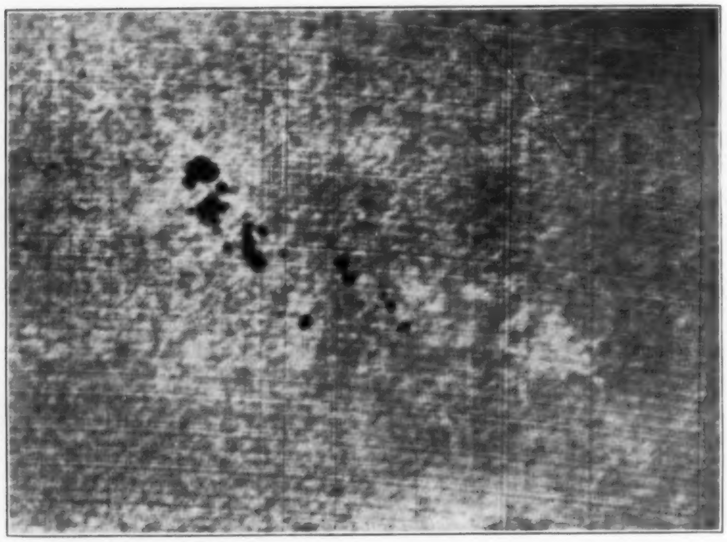
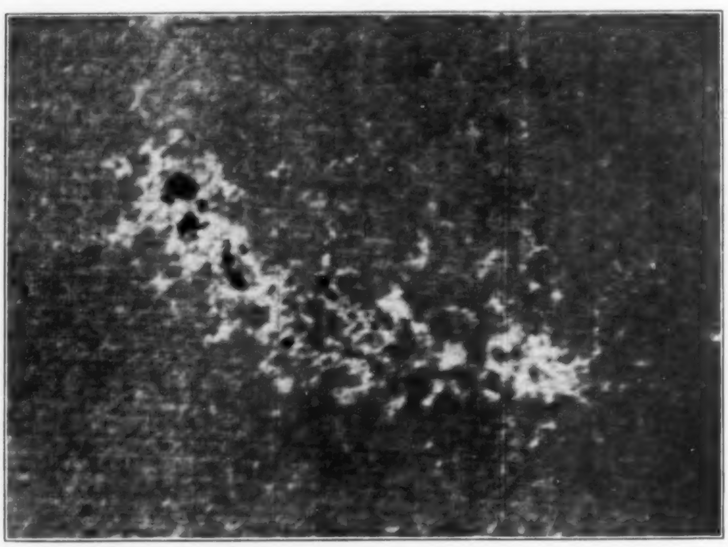


FIG. 1.—11^h 32^m. Faculæ. Slit on Continuous Spectrum at λ 3924.

E W



S

FIG. 2.—11^h 22^m. Calcium Flocculi, Low K_1 Level. Slit at λ 3929.

FACULÆ AND LOW-LEVEL SECTION OF CALCIUM FLOCCULI, 1903, APRIL 29.

(Scale: Sun's Diameter = 0.280 Meter.)

PLATE VII.

N

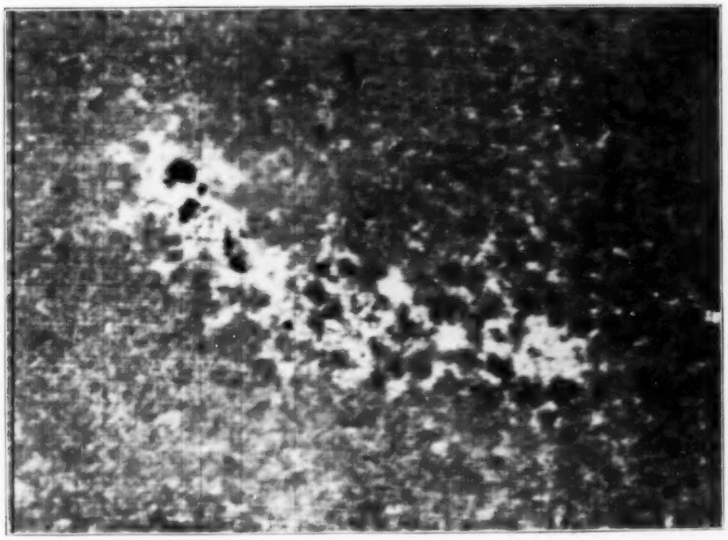
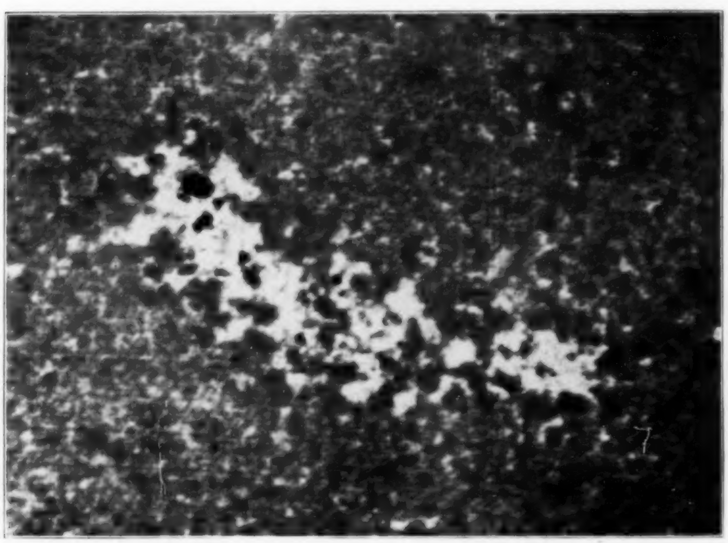


FIG. 1.—$10^h\ 28^m$. Calcium Flocculi, Higher K_1 Level. Slit at λ 3932.

E W



S

FIG. 2.—$11^h\ 11^m$. Calcium Flocculi, K_2 Level. Slit at λ 3933.8.

SECTIONS OF CALCIUM FLOCCULI AT DIFFERENT LEVELS, 1903, APRIL 29.

(Scale: Sun's Diameter = 0.280 Meter.)

which would seriously affect the photographs if not dispersed by the prisms before reaching the sensitive plate.

On developing the first plate made with the $H\beta$ line, we were surprised to find extensive *dark* regions, scattered over the surface of the Sun, and resembling in form the bright flocculi obtained with the H or K lines. On careful examination it was

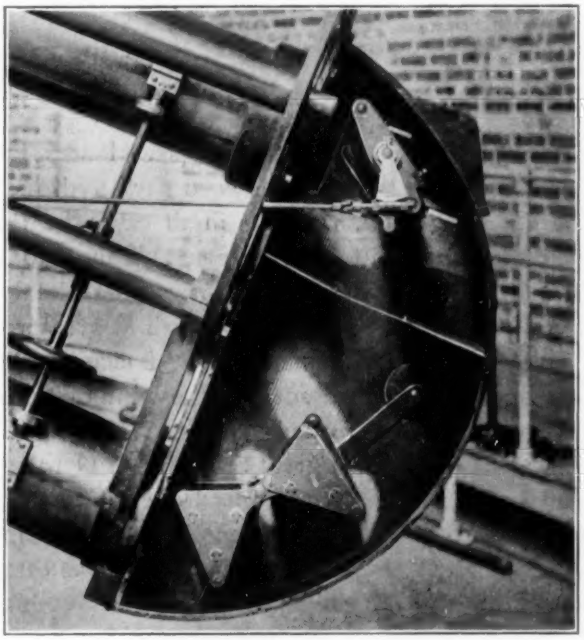


FIG. 1.—Prism-Box.

.ound that while the dark regions of hydrogen resembled in a general way the bright regions of calcium, nevertheless there was a marked difference in form in many instances, and almost never an exact agreement. The results given by this plate were confirmed by subsequent photographs, leaving no doubt that the hydrogen flocculi are, in general, dark, and that while they have a general resemblance in form to the bright calcium flocculi, the differences are in many cases very striking.

We soon found, as was expected from visual observations, that

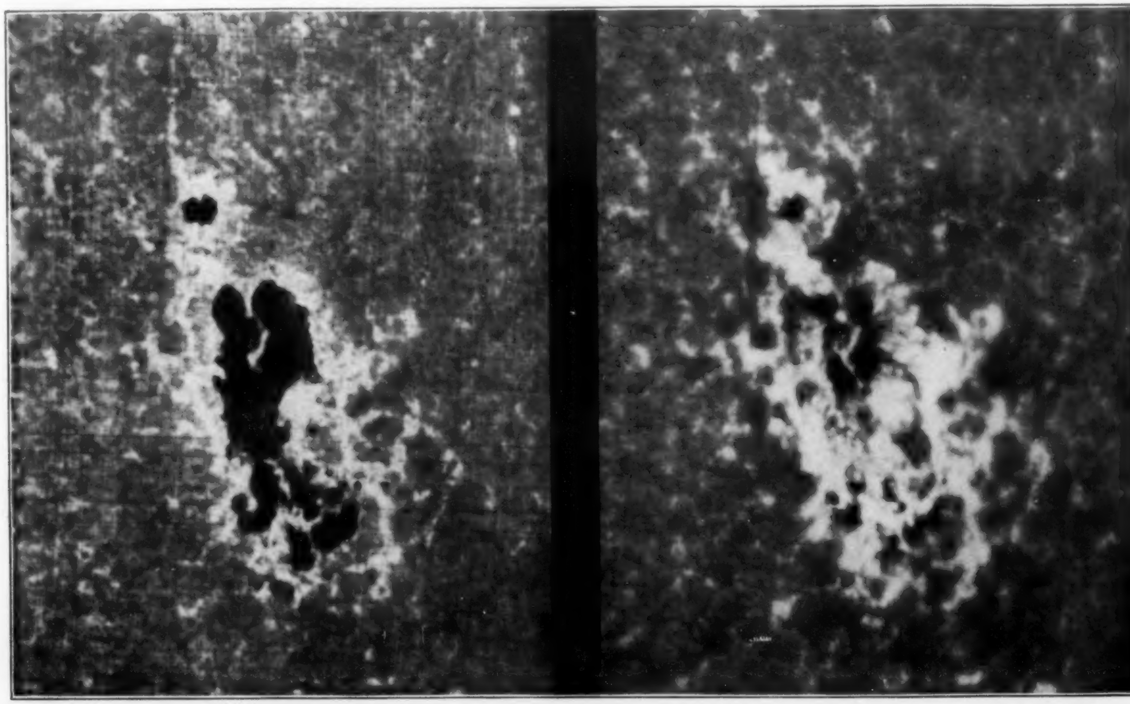
in the vicinity of active Sun-spots, and, in general, in regions of considerable activity, *bright* hydrogen flocculi often appear. While these are frequently eruptive in character, they are not always so. The temperature of the hydrogen gas (or whatever factor determines its radiating power) is apparently so near the neutral point that in some regions a flocculus may be dark, whereas, in an adjoining region, through superabundance of radiation, the flocculus may appear bright. Under such circumstances it is evident that the hydrogen which is exactly at the neutral point will not appear on the photographs. The hydrogen that does appear is shown because its radiating power is greater or less than that of the surrounding hydrogen gas. Plates IX to XII permit a comparison to be made between dark hydrogen flocculi and bright calcium flocculi, although in all cases the time interval is too great to make an accurate comparison of form of any value. In Fig. 2, Plate IX, it will be noticed that the flocculus is bright immediately about the spot, and dark at greater distances away. In Fig. 2, Plate X, small bright flocculi, indicative of eruptive phenomena, may be seen to the west of the spot. A more striking contrast is afforded by Plates XI and XII, which show the surroundings of the great Sun-spot of October 1903, as photographed with the calcium and hydrogen lines.

Photographs of the hydrogen flocculi have been made with the $H\beta$, $H\gamma$, and $H\delta$ lines. They are now taken as often as possible with the Rumford spectroheliograph, for systematic study in conjunction with the calcium photographs.

DARK CALCIUM FLOCCULI.

The discovery of the dark hydrogen flocculi led us to make a careful examination of our plates in the hope of finding conclusive evidence of the existence of *dark* calcium flocculi. We have found a number of cases, which seem to leave no doubt as to the reality of the phenomena. One of these is illustrated in Fig. 1, Plate XIII; the corresponding dark hydrogen flocculus is given for comparison (Fig. 2). It was through the exceptional intensity of this hydrogen flocculus that the dark calcium flocculus was found.

PLATE VIII.



October 9, $3^h\ 43^m$. Calcium Flocculi, Middle H_1 Level. Slit at $\lambda\,3966$.

October 9, $3^h\ 30^m$. Calcium Flocculi, H_2 Level. Slit at $\lambda\,3968.6$.

THE GREAT SUN-SPOT OF OCTOBER 1903.
For Comparison with the Stereoscope.

PLATE IX.

N

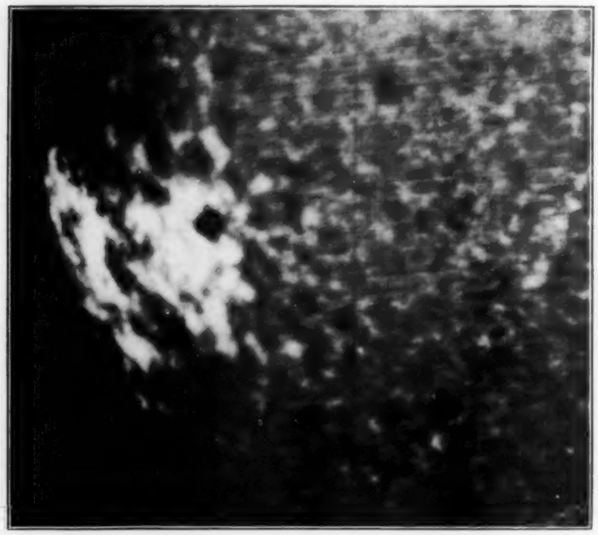
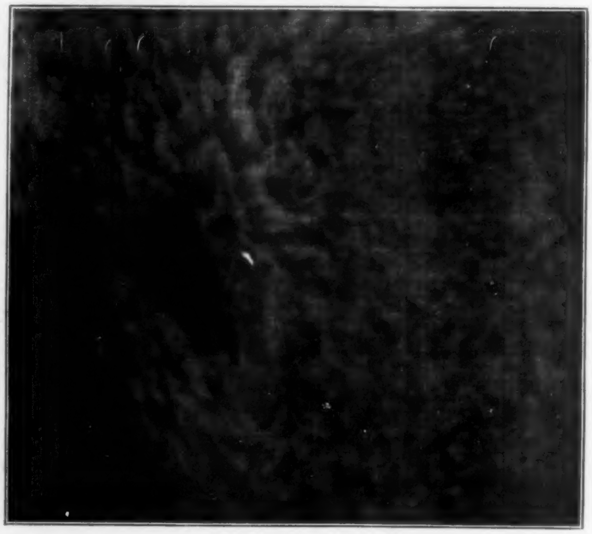


FIG. 1.—July 2, $9^h\ 39^m$. Calcium Flocculi, K_2 Level.
Slit at λ 3933.8.

E W



S

FIG. 2.—July 2, $3^h\ 40^m$. Hydrogen Flocculi.
Slit at Center of $H\beta$.

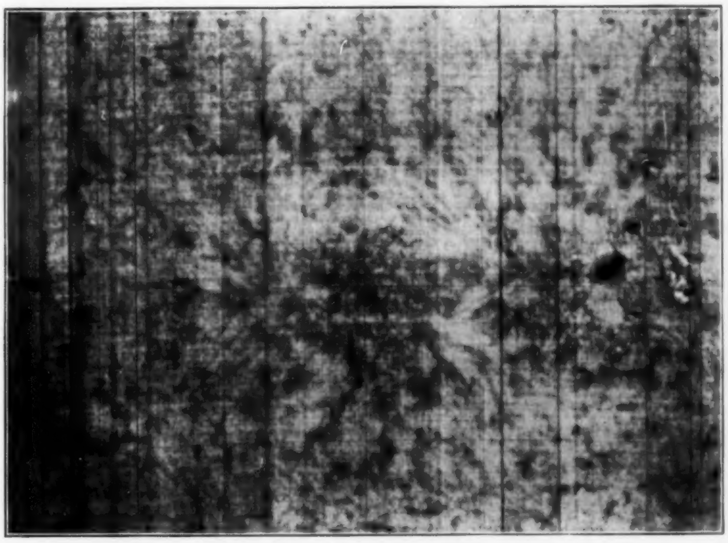
COMPARISON OF CALCIUM AND HYDROGEN FLOCCULI.
(Scale: Sun's Diameter = 0.335 Meter.)

PLATE X.

N

FIG. 1.—$3^h\ 57^m$. Calcium Flocculi, K_2 Level. Slit at λ 3933.8.

E W



S

FIG. 2.—$11^h\ 0^m$. Hydrogen Flocculi. Slit at Center of $H\gamma$. (Bright Eruptive Flocculi West of Spot.)

HYDROGEN AND CALCIUM FLOCCULI, 1903, JULY 7.

(Scale: Sun's Diameter = 0.290 Meter.)

FUTURE WORK.

The preliminary results here given will suffice to suggest the general character of the work which may be expected from the spectroheliograph in the future. If advances are to be made far beyond the point attained by the present work, it is evident that two conditions should be fulfilled: (1) Provision should be made for the use of spectroheliographs having much higher dispersion, so that some of the narrower dark lines may be employed. (2) Much advantage should be derived from the use of still larger solar images, especially at sites where the atmospheric conditions are particularly favorable. Unless the best seeing is available, it will be impossible to photograph the finest details of the solar surface. With a sufficiently large solar image, good seeing, and very high linear dispersion, it should become possible to make photographs showing the distribution of the vapors corresponding to the widened lines and the bright lines in Sun-spots. Such work should be of great importance, if carried on in conjunction with the study of the spectra of Sun-spots, made with instruments of high dispersion.

A report on the instrumental and atmospheric conditions required in future work on the Sun may be found in the forthcoming *Year Book* (No. 2) of the Carnegie Institution.

YERKES OBSERVATORY,
December 1903.

DESCRIPTION OF THE PLATES.

PLATE I.

Entire disk of Sun, as photographed 1903, August 12, $8^h\ 52^m$ C. S. T. with the H_2 line. Same size as original negative. The squares of the half-tone screen are too coarse to permit the smallest details to be shown.

PLATE II.

Minute calcium flocculi, H_2 level, showing their normal appearance under excellent conditions of seeing.

PLATE III.

H and K lines on the solar disk and in the chromosphere (radial slit). *a* shows H_3 and K_3 (very faintly) in a prominence.

PLATE IV.

FIG. 1.—The K line on the solar disk and in the chromosphere at the limb (radial slit). The bright reversals (K_2) are due to the flocculi. Where faculæ are present the continuous spectrum is more or less strengthened.

FIG. 2.—Minute calcium flocculi, resembling the granulation of the photosphere. The squares are 10″ of arc on a side.

FIG. 3.—Reversals of the H and K lines in the electric arc, showing the decrease in width from the inner (dense) to the outer (rare) calcium vapor.

PLATE V.

FIG. 1.—Low-level (H_1) section of calcium flocculi, showing how these flocculi appear to be made up of vertical columns of calcium vapor.

FIG. 2.—High-level (H_2) section of the same flocculi, showing (faintly) how the vapor columns seem to be bent over at the summit, as well as expanded.

PLATE VI.

FIG. 1.—For this photograph the second slit was set on the continuous spectrum at λ 3924. Consequently no flocculi are shown, though the faculæ are faintly visible. The forms of the latter should be compared with those of the flocculi in Fig. 2.

FIG. 2.—Low K_1 level. Slit set at λ 3929. This shows the dense calcium vapor not far above the photosphere. Compare with Fig. 1, and note that even at this low level the calcium vapor overhangs, and sometimes completely covers small spots.

PLATE VII.

FIG. 1.—Higher K_1 level. Slit at λ 3932. Though taken before the photographs reproduced in Figs. 1 and 2 of Plate VI, this picture further emphasizes the differences noted at lower levels. The fact that the changes are progressive largely eliminates the time element, which might otherwise be suspected of causing the observed differences. As a matter of fact, these flocculi are quiescent and slowly changing, differing very decidedly from eruptive phenomena.

FIG. 2.—K_2 level. Slit at λ 3933.8. Here the calcium vapor is very brilliant, and covers a larger area. The photograph contains distinct evidence of dark absorbing masses at higher levels. Perhaps the best instance of this is the dark tongue which runs somewhat north of west from the small spot south preceding the largest one of the group. This tongue seems to form a part of an extensive dark area, which completely surrounds the bright flocculi of the group.

PLATE VIII.

In comparing the photographs corresponding to different levels a double stereopticon is used, by which two negatives can be projected upon a screen, where the images are exactly superposed. The method has proved so instructive that it has seemed desirable to provide with this paper a simple

PLATE XI.

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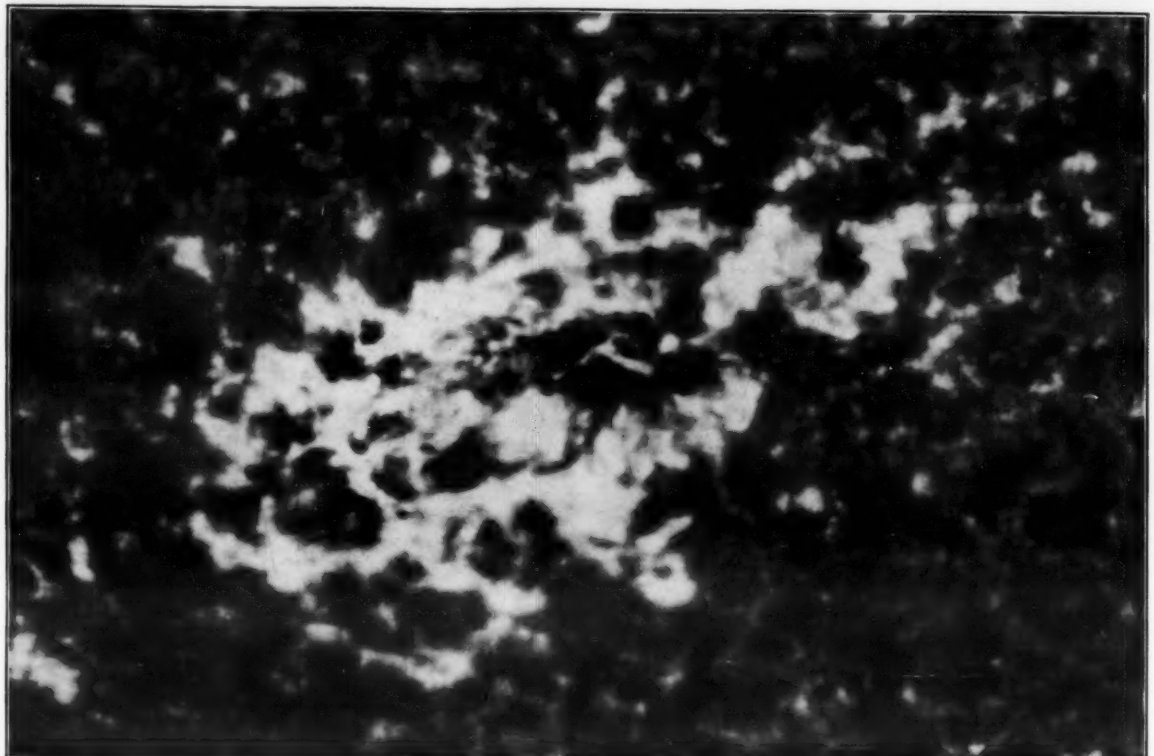
E



W

S

October 9, $3^h\ 30^m$. Calcium Flocculi, H_2 Level.

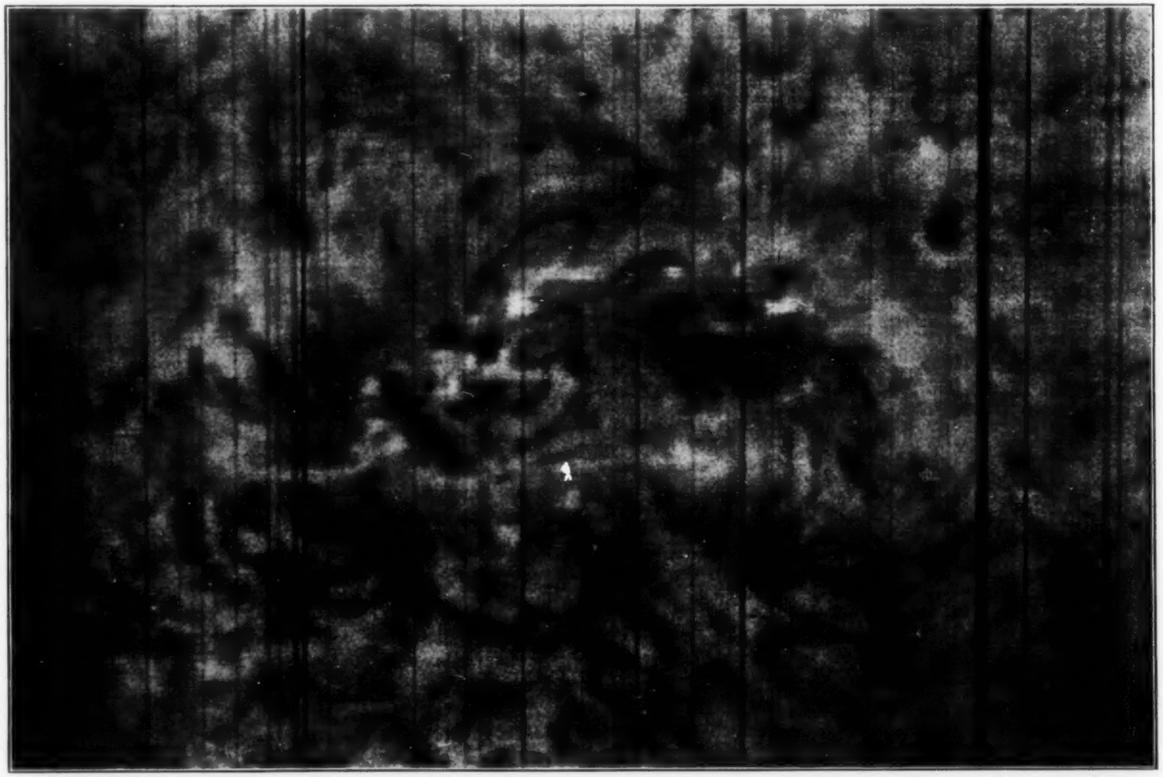
THE GREAT SUN-SPOT OF OCTOBER 1903.

(Scale: Sun's Diameter = 0.550 Meter.)

PLATE XII

N

E



W

S

October 9. $1^h\ 04^m$. Hydrogen Flocculi.

THE GREAT SUN-SPOT OF OCTOBER 1903

(Scale: Sun's Diameter = 0.550 Meter.)

means of accomplishing the same result. Accordingly, a pair of high- and low-level photographs has been arranged for use with the stereoscope. It is to be understood that no stereoscopic effect in the ordinary sense will be obtained in examining these photographs. The purpose of using the stereoscope is merely to allow the images to be superposed, thus permitting them to be seen at the same point in rapid succession by quickly moving a card so as to cover alternately the two lenses of the stereoscope. In this way the same region of the Sun may be examined, first as it appears at the low level of the denser calcium vapor, and then as it appears at the higher level of the rarer vapor. Thus the manner in which the calcium flocculi overhang the penumbra, and sometimes the umbra of spots, and the absence at the lower level of the dark structures shown in certain parts of the high-level picture, can be observed. This method of comparison also gives an excellent means of detecting small changes in the form of the flocculi, as shown by photographs corresponding to the same level, but taken at different times.

PLATE IX.

FIG. 1.—Calcium flocculi surrounding a spot when near the east limb of the Sun. The strong dark regions in this photograph are due to too great contrast in the original photograph, and not to dark calcium flocculi.

FIG. 2.—The same region about six hours later, as photographed with the $H\beta$ line. Near the spot the hydrogen immediately surrounding the spot appears to be bright, while an extensive dark hydrogen flocculus lies to the east, occupying approximately the same region as that of the bright calcium flocculus.

PLATE X.

FIG. 1.—The contrast here is rather too great, and for this reason the background appears too dark. The general character of the bright calcium flocculi is nevertheless fairly well shown.

FIG. 2.—The contrast in this photograph is more nearly what it should be, though the background is in general too bright. Some well-defined examples of bright hydrogen flocculi may be seen to the west of the spot, where small spots were developing at the time.

PLATE XI.

At this level the penumbra is almost completely covered by the flocculi, while many of the smaller spots are blotted out. There are also distinct evidences of dark flocculi, due to absorbing vapors at still higher levels. The illustration necessarily fails to indicate the brilliancy of the brightest eruptive phenomena, which on the original negative are easily distinguished from the ordinary flocculi.

PLATE XII.

This photograph, which shows the hydrogen flocculi surrounding the spot group, should be compared with Plate XI. The brighter regions are in most

cases eruptive. In general the hydrogen flocculi in the less disturbed regions are dark, though they may perhaps be bright or neutral where they overhang the penumbra, and cover some of the smaller spots of the group.

PLATE XIII.

FIG. 1.— Dark calcium flocculus, corresponding to the exceptionally dark hydrogen flocculus shown in Fig. 2, which represents the same region of the Sun. The contrast in Fig. 1 is rather too great, and some regions which might seem to resemble dark flocculi should appear much lighter.

FIG. 2.—Exceptionally strong dark hydrogen flocculus. Compare with Fig. 1.

It should be noted that all the photographs of hydrogen flocculi are slightly distorted, owing to the fact that a straight first slit and a curved second slit are used with the grating.

ON METHODS OF TESTING OPTICAL MIRRORS DURING CONSTRUCTION.

By G. W. RITCHEY.

NO GREATER mistake could be made than to assume that cheap and poorly annealed disks of glass, or those with large striæ or pouring marks, are good enough for mirrors of reflecting telescopes. While I am not prepared to say that optical glass of the finest quality must be used for mirrors to secure the best attainable results, it is evident that a high degree of homogeneity and freedom from strain is necessary in order that the figure of a mirror shall not be injuriously affected by temperature changes. If it were not necessary to consider the question of cost, I should advise the use of the finest optical (crown) glass for mirrors, in order to be as free as possible from risk. Usually considerations of expense would, in the case of large mirrors, make it necessary to chose between an optical disk of a given size, and a somewhat larger one of the kind furnished by the St. Gobain Company, for example. The diagonal plane mirror for a Newtonian, and the convex mirror of the Cassegrain reflector, should always be made of the best optical glass, since the expense for these is comparatively slight.

A very important point is in regard to the necessary thickness of the glass disks. As a result of experience with many mirrors of from six to sixty inches diameter, in which the thickness of the several disks varies from one-twelfth to one-sixth of the diameter, I have no doubt that the thicker disks are always preferable, provided that they are as homogeneous and as well annealed as the thinner ones. The thinner mirrors suffer much greater temporary change of curvature from the very slight heat generated during the process of polishing, and they undoubtedly suffer greater temporary disturbance of figure from changes of temperature when in use in the telescope. In the case of the large paraboloidal mirror of a reflecting telescope, which should always be properly supported at the

back to prevent flexure, the thickness should be not less than one-eighth or one-seventh of the diameter; in the writer's opinion the latter ratio leaves nothing to be desired. In the cases of the small diagonal mirror and of the small convex mirror, which cannot easily be supported at the back, the thickness should be not less than one-sixth of the diameter.

All mirrors should be polished (not figured) and silvered on the back as well as on the face, in order that both sides shall be similarly affected by temperature changes when the mirrors are in use in the telescope; for the same reason the method of supporting the large mirror at the back, in its cell, should be such that the back is as fully exposed to the air as possible.

The following is a brief description of the methods used by the writer in the optical laboratory of the Yerkes Observatory, in testing spherical, plane, paraboloidal, and hyperboloidal mirrors, while they are being figured. The methods of grinding, polishing, and figuring are not described here; these methods, and other matters relating to optical work, to the proper support of mirrors in their cells, and to the mountings of reflecting telescopes, will be described in a paper to be published in 1904 by the Smithsonian Institution.

In all cases the methods given enable the optician to test the mirror surfaces in a rigorous manner, as a whole, *i. e.*, without the use of zonal diaphragms or local tests of any kind. A great saving of time is thus effected; for testing should be done at frequent intervals during the work of figuring; frequently a mirror is tested a hundred times or more, as its figure is gradually improved. The tests are all conducted in the optical laboratory, which must be so arranged that temperature and atmospheric conditions are under perfect control. It is impossible to overestimate the importance of being able to test quickly, as often as is desired, and under conditions of constant temperature and quiet air—conditions which are seldom realized in a dome or out of doors. All mirrors when being tested are placed on edge, so that the axis of figure is nearly horizontal, large mirrors being suspended in a wide, flexible steel band lined with soft paper or Brussels carpet.

I. CONCAVE SPHERICAL MIRRORS.

The method of testing concave spherical mirrors is well known; it has been admirably described by Foucault, Draper, Common, and others. It is described briefly here, as it forms a necessary introduction to the tests of other forms of surfaces. A small, brilliant source of light, or "artificial star," may be produced by placing in front of the flame of an oil lamp a thin metal plate in which a very small pinhole has been bored. If the illuminated pinhole be placed about an inch to one side of the principal axis of the mirror, and at a distance from the mirror equal to its radius of curvature, a reflected image of the pinhole will be formed on the other side of the axis and at the same distance from it and from the mirror as the corresponding distances of the pinhole itself. If the surface of the mirror is perfectly spherical and there be no atmospheric disturbances in the course of the rays, the reflected image, when examined with an eyepiece, will be found to be a perfect reproduction of the pinhole, with the addition of one or more diffraction rings around it, minute details of the edges of the pinhole appearing as exquisitely sharp and distinct as when the pinhole is itself examined with the eyepiece. If the eyepiece be moved outside and inside the focus, the expanded disk in both cases appears perfectly round. Nothing can be more impressive than to see such a reflected image produced by a fine spherical mirror having a radius of curvature of one hundred feet or more. Several such mirrors of two feet aperture have recently been finished here.

The use of an eyepiece is interesting for such experiments as that just described, and is important as a check upon the test with the opaque screen. The latter test, however, which I shall call the knife-edge test, is used almost exclusively with mirrors of all forms; it is far more serviceable than the eyepiece test in determining the nature and position of zonal irregularities, and is far more accurate in determining the radius of curvature either of a mirror as a whole or of any zones of its surface.

If the eye be placed just behind the reflected image of the illuminated pinhole, so that the entire reflected cone of light

enters the pupil, the polished, unsilvered mirror surface is seen as a brilliant disk of light. Let an opaque screen or knife-edge be placed in the same plane through the axis as the pinhole, and be moved across the reflected cone *from the left* and just in front of the eye; if a dark shadow is seen to advance across the mirror from the left, the pinhole and knife-edge are inside of the best focus and must be moved together away from the mirror; if, however, with the screen still moved across from the left, the shadow advances across the mirror from the right, pinhole and screen are outside of the focus and must be moved toward the mirror. By repeated trials a position is found in which the shadow does not appear to advance from either side, but the surface of the mirror darkens more or less uniformly all over; this is the position or plane of the best focus, and it is with this position of the knife-edge that irregularities of the surface, if any exist, are seen in most highly exaggerated relief; with this position of the knife-edge, the mirror, if perfectly spherical, is seen to darken with absolute uniformity all over, as the screen is moved across the focus, and the impression is given to the eye of a perfectly plane surface.

If, however, the mirror surface is not perfectly spherical, but contains several zones of slightly different radii of curvature—a very common case—the zones will appear as protuberant or depressed rings on an otherwise plane surface. The reason for this is evident; the light from some parts of such zones is cut off by the knife-edge before, from other parts after, the illumination from the general surface is cut off. The surface is therefore seen in light and shade, *i. e.*, in enormously exaggerated relief. The mirror must be regarded as being illuminated by light shining very obliquely along the surface from the side opposite that from which the knife-edge advances across the focus. The interpretation of the lights and shades becomes easy after a little experience; not only is the character of a zone—whether it be an elevation or depression—readily seen, but its diameter and its width are readily determined.

If the disk of glass is of sufficient thickness and of proper quality, and if attention has been given to the uniform rotation

of the turntable on which the mirror is polished, and to the protection of the glass from abnormal conditions of temperature during grinding and polishing, all irregularities of figure which occur are perfect zones or rings concentric with the edge of the glass; that is, the surface is always a perfect surface of revolution. If, however, the disk is too thin, or has been improperly supported during grinding and polishing, or if it has been cut out of thick rolled plate glass so that it is weak in the direction of one diameter, an astigmatic mirror may be produced, in which the radius of curvature is slightly different along two diameters at right angles to each other.

Astigmatism is easily recognized with either the knife-edge or the eyepiece test. Let the plane of the apparent focus be determined with the knife-edge advancing from the left, then from above, then from the right, then from a number of directions between these three; if astigmatism exists, the planes of the various foci thus found will not coincide; and the directions of greatest and least curvature of the surface are readily determined. When the eyepiece test is used, an astigmatic mirror does not give a sharp image even at the best focus; if the eyepiece be moved outside or inside of this focus, the expanded disk becomes elongated and is not uniformly illuminated; the direction of elongation outside is at right angles to that inside, and the distribution of light in the expanded disk is entirely different outside and inside of the focus.

The general character of the tests having been described, let us consider some important matters of detail which are necessary for the greatest refinement in testing all forms of mirrors.

By the use of a small lens and a diagonal prism, in the manner shown in Fig. 1, the lamp may be kept well out of the way, and the illuminated pinhole and its reflected image brought very close to the axis of figure of the mirror. This is of much importance in testing mirrors of short focus or of great angular aperture, as the danger of errors in testing due to working considerably out of the axis of figure is avoided. As may be seen in the figure, the pinhole is now placed at the surface of the diagonal prism nearest to the mirror being tested. The arrange-

ment should be such that the angle of the cone of rays proceed ing from the lens is considerably larger than is needed to fill the concave mirror.

During the work of figuring, mirrors are usually tested while unsilvered, since very frequent tests are desirable. While the amount of light reflected from the polished unsilvered surface is surprisingly great, a much more brilliant artificial star than that given by the oil lamp is required for the greatest refinement and accuracy with the knife-edge test, especially in the cases of plane, paraboloidal and hyperboloidal mirrors, in which there are two reflections from the unsilvered surface. It might be

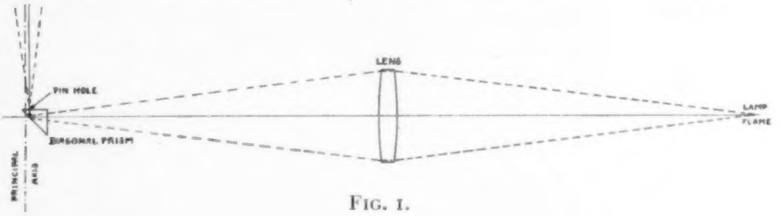


FIG. 1.

supposed that a larger pinhole could be used, and thus a more brilliant illumination of the mirror surface secured; but a large pinhole allows an apparent diffusion of light over the mirror surface, which obliterates all of the more delicate contrasts of illumination, due to minute irregularities of surface. With feeble illumination of the mirror surface, the eye is entirely unable to detect slight contrasts which with brilliant illumination become strong and unmistakable. When the knife-edge test is used with an extremely small pinhole, of between 0.05 mm and 0.1 mm diameter, illuminated by acetylene, or (what is much better) oxyhydrogen or electric arc light, minute zonal irregularities are strongly and brilliantly shown, which are entirely invisible with a larger pinhole or insufficient illumination. With the arrangement of lens and diagonal prism (Fig. 1), either of the sources of light named can be used without difficulty; disturbances of the air from their heat should be prevented by placing the light behind a partition with a window of thin plate glass.

With the best conditions of apparatus just described the degree

of accuracy to be attained with the knife-edge test is surprising. With a spherical mirror of two feet aperture and fifty feet radius of curvature, the plane of the center of curvature can be easily located to within 0.01 inch (0.25mm), and, with care, to within half of that amount. With the dimensions given, a change of 0.01 inch in the radius of curvature corresponds to a change of $\frac{1}{500000}$ inch (0.00005mm) in the depth of the curve of the mirror surface. There can be no doubt that zonal irregularities of surface of half this amount are readily recognized.

A large and perfect spherical mirror is an indispensable part of the equipment of an optical laboratory, as it affords what is, in my opinion, the most satisfactory means of testing large plane mirrors. In making a paraboloidal mirror, also, it is best first to produce a spherical surface of the proper radius of curvature and then to change gradually to the paraboloid.

II. PLANE MIRRORS.

The making of large plane mirrors of fine figure is usually regarded as much more difficult than the making of large concave mirrors. The difficulty has been, in the past, largely one of testing. With a satisfactory method of testing a large plane surface *as a whole*, in a rigorous and direct manner, the problem is greatly simplified. So far as the writer is aware, no such test has hitherto been fully developed. In *Monthly Notices*, **48**, 3, 1887, Mr. Common suggests, very briefly, the testing of plane mirrors in combination with a finished spherical one, and uses a diagram in illustration; but no details in regard to the method are given. This method has been developed and used for many years by the writer, in testing plane mirrors up to thirty inches in diameter. When this test is used, the difficulty of making a twenty-four-inch plane mirror which shall not deviate from perfect flatness by an amount greater than $\frac{1}{500000}$ inch (to use an example similar to that given in the preceding paragraph) is neither greater nor less than that of making a good spherical mirror of two feet aperture and fifty feet radius of curvature, when it is required that the radius of curvature shall not differ from fifty feet by a quantity greater than $\frac{1}{100}$ inch.

A spherical mirror *A* (Fig. 2) which should not be smaller in diameter than the plane mirror *B*, to be tested, is figured with the utmost accuracy, special care being taken that no astigmatism, however slight, exists in it. The mirror *A* is silvered; *B* is polished and unsilvered. The mirrors may be set up as shown *in plan* in Fig. 2, the distance $cm + mf$ being equal to the radius of curvature of *A*; both mirrors hang on edge in steel bands, as already described. The light from the illuminated pinhole strikes *B*, is reflected to *A*, thence back to *B*, thence to a focus close beside the illuminated pinhole.

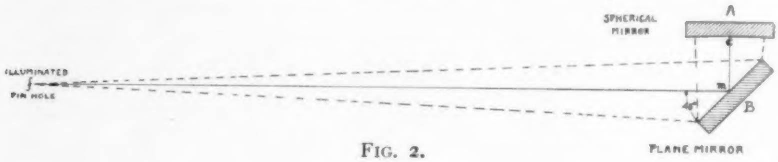


FIG. 2.

When using the knife-edge test the optician sees the mirror *B* brilliantly illuminated and in elliptical outline, the horizontal diameter appearing foreshortened by an amount depending upon the angle at which the mirror is viewed. With the knife-edge test the surface of *B* is seen in relief, as a whole; any zonal errors appear enormously exaggerated, and their character and position are readily determined, just as when a spherical mirror is tested at its center of curvature; these zonal errors of course appear elliptical on account of their foreshortening; their effect is doubled in intensity on account of the two reflections from *B* (assuming that the illumination is as brilliant as the eye requires).

The test as already described is all that is necessary for the detection and location of zonal errors. But something more is needed in order to detect general curvature, *i. e.*, general convexity or concavity in *B*. It is assumed that the mirror, when fine-ground and polished, is so nearly flat that no curvature can be detected with a Brown & Sharpe steel straight-edge of the finest quality. For convenience in description let us also assume that the surface is free from zonal errors. Let the knife-edge be moved across the reflected cone from the left; a focal point

will be found at which the right and left sides of the mirror darken simultaneously; this focal point we will call f_1. Now let the knife-edge be moved across the cone from above, instead of from the left; a focal point will be found at which the upper and lower parts of the mirror darken simultaneously; this focal point we will call f_2. It is only when the mirror B is a perfect plane that f_1 and f_2 coincide with each other and with the point f (see figure). If B is slightly convex, both f_1 and f_2 are outside of f (*i. e.*, farther from the mirror than f), and f_1 is outside of f_2; if B is slightly concave, both f_1 and f_2 are inside of f, and f_1 is

FIG. 3.

inside of f_2. In practice the exact position of f is not found, except incidentally when the plane mirror is finished, for this would involve the very accurate measurement of the large distance $cm + mf$. The determination of the positions of f_1 and f_2 with respect to each other is all that is needed.

That f_1 and f_2 do not coincide when B is convex or concave is due to the fact that the curvature of B is apparently increased or exaggerated in the direction of the horizontal diameter of the mirror, on account of its foreshortening in this direction as seen from f; while the curvature in the direction of its vertical diameter is not thus exaggerated. The effect is precisely as if the spherical mirror A were astigmatic, the parts of the surface adjacent to the horizontal diameter having a different radius of curvature from those adjacent to the vertical diameter. This effect is so marked that an extremely small deviation of B from a true plane can be detected. For example, if A and B are each two feet in diameter, the radius of curvature of A being fifty feet as before, and if the angle which the line fm subtends with the surface of B is 45°, a departure from a true plane of $\frac{1}{350000}$ inch (0.00007mm) in the surface of B is readily detected. If the angle of the mirror B be changed to 30°, as shown in Fig. 3, the

accuracy of the test for general curvature is about doubled; the latter position, however, is not usually so convenient for determining the positions of zonal irregularities; for the greatest refinement, therefore, the stand on which *A* and *B* are supported is so designed that the positions of the mirrors can be quickly changed so as to give the greatest accuracy in each part of the test.

The use of an eyepiece in this test is important because it shows how fatal to good definition is even a very slight convexity or concavity of a plane mirror when used in an oblique position. If f_1 and f_2 are made to coincide as closely as can be detected with the knife-edge test (*B* being free from zonal errors also) the reflected image of the pinhole, as seen with an eyepiece at f_1 is as exquisitely sharp and perfect as if it were formed by the spherical mirror *A* alone. But if *B* is very slightly convex or concave, the appearance of the eyepiece image is similar to that which has already been described in connection with astigmatic concave mirrors.

III. PARABOLOIDAL MIRRORS.

The methods of testing paraboloidal mirrors at their center of curvature, by determining the radius of curvature of successive zones, was brought to a state of great refinement by Draper, Common, and others. These workers used an eyepiece in determining the foci of the various zones. In the ASTROPHYSICAL JOURNAL for November 1901, **14**, 218, the writer described a modification of this test, in which the knife-edge instead of an eyepiece is used in determining the foci; with this modification very narrow zones or arcs can be used, and much greater accuracy attained in testing mirrors of large angular aperture, in which the curvature of the surface changes rapidly. In Fig. 4 is shown the diaphragm with which the successive zones were exposed in testing in this manner the large mirror of the two-foot reflector. The illuminated pinhole remains fixed in the plane of the center of curvature of the central parts of the paraboloid, *i.e.*, at a distance 2 *F* from the vertex, where *F* is the focal length; the plane in which the knife-edge must be placed to cause the right and left sides of a given zone to darken simultaneously is the plane

in which the rays reflected from this zone are brought to a focus. When the paraboloidal figure is perfect, the rays reflected from any very narrow zone whose semi-diameter is R are brought to a focus at a distance $\frac{R^2}{2F}+\frac{R^4}{16F^3}$ back of the plane of the pinhole; *i. e.*, at a distance $2F+\frac{R^2}{2F}+\frac{R^4}{16F^3}$ from the vertex of the paraboloid.

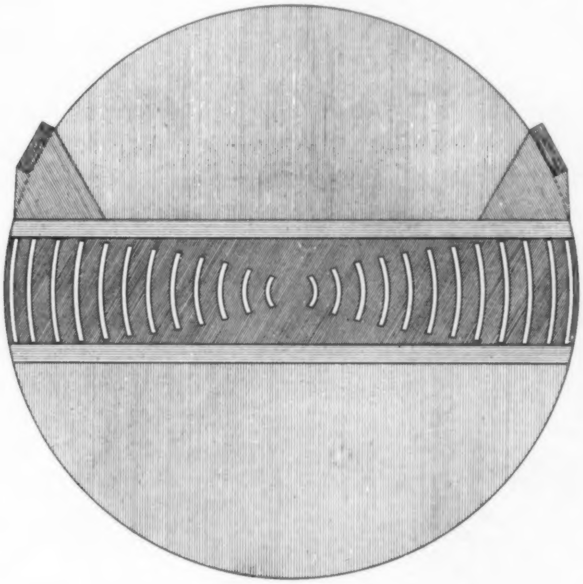


FIG. 4.

But when testing is done at the center of curvature, even with the extremely accurate method just described, the making of a large paraboloidal mirror of great angular aperture and really fine figure is an exceedingly difficult task. This is due in part to the necessity of very frequent tests, in each of which the foci of a large number of zones must be determined; it is due even more to the uncertainty in determining the exact nature of errors of surface (considering the surface as a whole), corresponding to focal readings which do not agree with the computed values. In the case of mirrors of small or moderate angular aperture much important information can be gained by viewing the surface as a

whole from the (mean) center of curvature, by means of the knife-edge test; a finished paraboloid, when thus seen, appears to stand out in relief, in strong light and shade, as a surface of revolution whose section is that shown in Fig. 5. Knife-edge and pinhole are both in the plane of the center of curvature of the zone *a;* the apparent curve should be a perfectly smooth one. But in the case of a mirror of large angular aperture the change of curvature is so rapid that only a narrow zone can be well seen at one time, *i.e.*, with a given focal setting of the knife-edge.

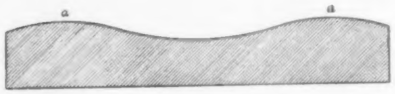


FIG. 5.

Attention has been called to the method of testing paraboloids at the center of curvature in order to contrast it with that next described, which is incomparably more simple, direct, and rigorous. The paraboloid is tested at its *focus* instead of its center of curvature. A brief note in regard to this method has already been published in this JOURNAL[1] by the writer. A well-figured plane mirror, which must not be smaller than the paraboloidal

FIG. 6.

one, is necessary in order that the testing may be done in the optical laboratory. In practice a small diagonal plane is also used, to avoid the necessity of a hole through the center of the large plane. Both of the plane mirrors are silvered. The arrangement of mirrors is shown in Fig. 6. The diagonal prism is placed at *f*, with the illuminated pinhole very near to the axis; pinhole and knife-edge are in the same plane, at a distance from

[1] 14, 218, 1901.

the vertex equal to $cm + mf$, which is equal to the focal length of the mirror. The paraboloid is now tested as a whole, without the use of zones, precisely as a spherical mirror is tested at its center of curvature.

If F be the desired focal length of the paraboloidal mirror whose semi-diameter is R, then the spherical surface, which is fine-ground and fully polished preparatory to parabolizing, should have a radius of curvature of $2F + \frac{R^2}{4F}$. This is because parabolizing is done by shortening the radii of curvature of all the inner zones of the mirror, leaving the outermost zone unchanged, as shown in Fig. 7; this is a far easier and better method in practice than to leave the central parts of the mirror unchanged and to lengthen the radii of curvature of all the other zones, as shown in Fig. 8.

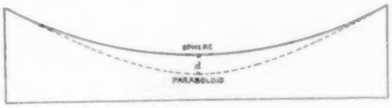


FIG. 7.

FIG. 8.

Let us now suppose that the concave mirror shown in Fig. 6 is spherical with radius of curvature $2F + \frac{R^2}{4F}$, where R is the semi-diameter and F is the distance $cm + mf$ from the center of the mirror surface to the plane of the pinhole and knife-edge. If the spherical surface be now viewed from the point f with the knife-edge test, it will appear to stand out in relief, in strong light and shade, as a surface of revolution whose section is shown in Fig. 9, the height of the protuberant center depending upon the angular aperture of the mirror.


FIG. 9.

The reason for this appearance is readily seen by reference to Fig. 7. To change the spherical surface to a paraboloidal one, the protuberant center must be removed by the use of suitable polishing tools, until the surface, as seen with the knife-edge test from the point f, appears perfectly flat, *i. e.*, the illuminated surface darkens with perfect uniformity all over. As the para-

boloidal surface nears completion, an elevated or depressed center, a "turned-up" or "turned-down" edge, or protuberant or depressed zones, can be seen, and their character and exact position be determined, with precisely the same ease and certainty with which similar irregularities are seen when a spherical mirror is examined at its center of curvature with the knife-edge test.

The actual difference of depth at the center or vertex, corresponding to *d*, Fig. 7, between a paraboloid and the nearest spherical surface, is small for such angular apertures as are used in practice. In the case of the mirror of the two-foot reflector, the focal length of which is only 236 cm (93 inches), the difference *d* is almost exactly 0.01 mm or 0.0004 inch; it is unusually large in this case on account of the exceptionally great ratio of aperture to focal length. The quantity *d* varies as the fourth power of the diameter of the mirror and inversely as the cube of the focal length. In the case of Lord Rosse's mirror of six feet aperture and 54 feet focus (ratio 1 to 9) the corresponding difference is only 0.0001 inch, very nearly; while in the case of the five-foot mirror of the Yerkes Observatory of twenty-five feet (762 cm) focus this difference is about 0.0006 inch (0.015 mm).

It should be noticed that even when the pinhole and reflected image are very near each other, as they should be, both may be far out of the axis of the paraboloid, if the mirrors are not perfectly adjusted or collimated; when this is the case, the mirror surface, when seen with the knife-edge test, does not appear as a surface of revolution, and cannot be properly tested. There are several simple methods of collimating the mirrors, thus insuring that the pinhole and the reflected image are both extremely near the optical axis; these methods need not be described here. The frame which supports the concave mirror can readily be so designed that this mirror can be removed and replaced repeatedly, while figuring it, without sensibly disturbing the adjustments.

The difficulties of making short-focus paraboloidal mirrors of fine figure are so greatly reduced when this method of testing is used that I believe that the general adoption of this method by

opticians would lead to such an improvement in results as to bring about a marked advance in the usefulness of reflecting telescopes. The making of the large plane mirror which is necessary in this test becomes so simple and certain when the method of testing described earlier in this article is used that I have no hesitation in saying that when a large paraboloidal mirror of short focus and of the finest attainable figure is to be made, it is economical to make a plane mirror of the same size with which to test it, if one is not already available. The concave mirror is first figured spherical, and is used thus in testing the plane mirror while the latter is being figured; the plane mirror is then used in testing the concave one during the parabolizing of the latter. Both the plane and paraboloidal mirrors are then used in testing the (convex) hyperboloidal mirror while the latter is being figured.

IV. HYPERBOLOIDAL MIRRORS.

The methods of figuring and rigorously testing convex hyperboloidal mirrors are now so thoroughly developed that the reflecting telescope can be regarded as a universal photographic telescope of the highest class, capable of giving, at the focus of the paraboloidal mirror of large angular aperture, the finest photographs now attainable of large and excessively faint objects such as the nebulæ in general; while by the addition of the small convex mirror a great equivalent focal length is obtained for the photography of bright celestial objects requiring large scale, such as the Moon, the planets, the dense globular star-clusters, and the annular and planetary nebulæ. The convex mirror of course serves as an amplifier, and possesses the great advantages over a lens used for this purpose that the perfect achromatism and the high photographic efficiency of the reflector are retained, and that the mechanical arrangements are very compact and economical. In order to give perfect definition the convex mirror must be an hyperboloidal one.

Fig. 10 shows the arrangement of mirrors employed in the two-foot reflector when used as a Cassegrain, a small diagonal plane mirror being used at *m*. *P* is the paraboloidal mirror with

its focus at f; H is the hyperboloidal mirror; the secondary focus or magnified image produced by the combination being at F; the point c is the center of the hyperboloidal surface. Calling the distance $fc = p$, and the distance $cm + mF = p'$, then $\frac{p'}{p}$ rep-

FIG. 10.

resents the amount of amplification introduced by the convex mirror. The radius of curvature R of the spherical surface, to which the convex mirror is ground and polished preparatory to hyperbolizing, is found with sufficient accuracy for all practical purposes by the formula $\frac{1}{p} - \frac{1}{p'} = \frac{2}{R}$, whence $R = \frac{2pp'}{p' - p}$.

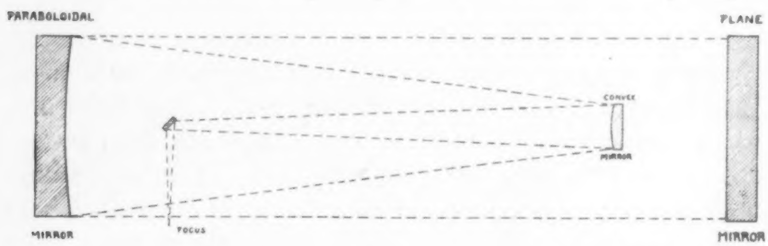


FIG. 11.

The method of testing the convex mirror while hyperbolizing it is shown in Fig. 11. The illuminated pinhole is placed very near the axis at F. The diverging cone of light strikes the small plane mirror, then the convex, then the large paraboloid, whence, if all the mirrors are finished and are well adjusted or collimated, the light is reflected in a parallel beam to the large plane; returning, the rays are brought to a focus very near the axis of figure and in the plane of the illuminated pinhole. All of the mirrors except the convex one are silvered. The convex spherical surface with radius of curvature R, as above described,

PLATE XIII.

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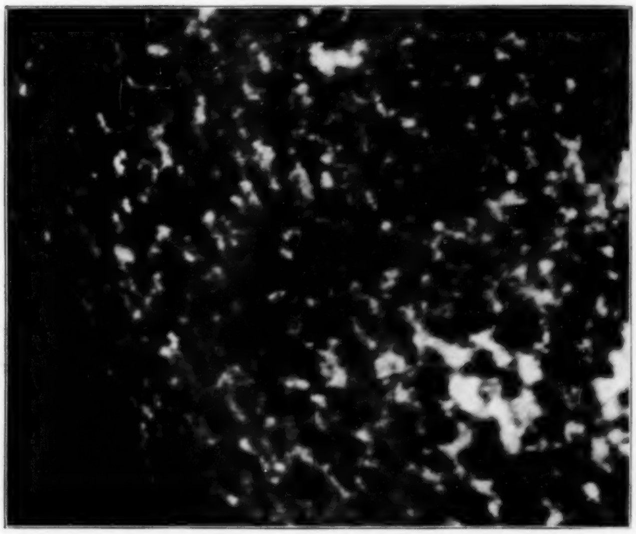

FIG. 1.—June 17, 9^h 08^m. Dark Calcium Flocculus. Slit at λ 3933.8.

E W

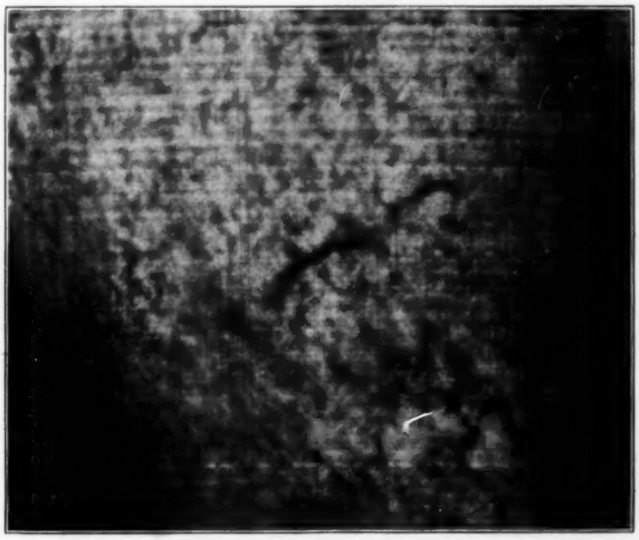

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FIG. 2.—June 17, 10^h 18^m. Strong Hydrogen Flocculus.
Slit at Center of $H\beta$. (Compare with Fig. 1.)

COMPARISON OF DARK CALCIUM AND HYDROGEN FLOCCULI.
(Scale: Sun's Diameter = 0.350 Meter.)

when viewed with the knife-edge test from the point *F*, presents the same general appearance of a smoothly curved surface of revolution, in strong light and shade, which a paraboloidal surface presents when similarly viewed from its center of curvature (see Fig. 5). All that is necessary to produce the hyperboloidal surface is to soften down, with suitable polishing tools, the apparent broad protuberant zone between the center and edge, until the mirror, as seen from *F*, appears perfectly flat; *i. e.*, until the illuminated surface is seen to darken with absolute uniformity all over when the knife-edge is moved across the focus.

As in the case of the paraboloid, it is necessary in this test that all of the mirrors be lined up or collimated with care; otherwise the surface of the convex mirror will not appear as a surface of revolution, and cannot be properly tested. The axes of the paraboloid and hyperboloid must coincide, and the face of the large plane mirror must be at right angles to these axes. In practice these adjustments are not difficult, and when they are once effected the convex mirror can be removed from its cell and replaced, while being figured, without sensibly disturbing them.

Yerkes Observatory,
December 10, 1903.

MICHELSON'S THEORY OF THE DISPLACEMENT OF SPECTRAL LINES.[1]

By J. Fényi.

Professor Wladimir Michelson has very recently given an entirely new and trustworthy explanation of the displacement of spectral lines on the Sun, which is distinguished from the bold hypotheses advanced in recent years by the facts that it is not only based upon an unassailable scientific foundation, but also that it finds an astonishing confirmation in the observations of these remarkable phenomena. Michelson shows that a displacement of lines must occur when a denser mass is introduced in the path of a ray of light. I will first give here another derivation of the appropriate formulæ, in order to make possible a test of the applicability of this theory, which is of such importance

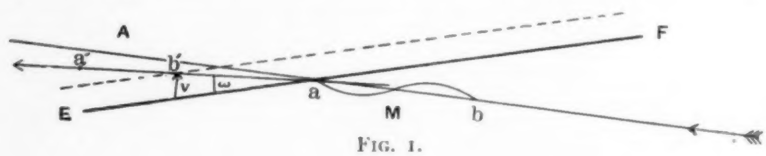

Fig. 1.

for the correct interpretation of solar phenomena. In the above figure the line $E\ F$ denotes the surface of separation of the mass M, at the right, from the ether A, at the left; a and b represent the length of a light-wave emerging into the ether from the mass, supposed at rest. As the end of the wave emerges at the point a, its front will have advanced in the ether to a'; the wave-length in ether, which we will designate by λ_0, is

$$\lambda_0 = aa' = abn \ ,$$

where n is the index of refraction. The color of the light then remains unchanged, and the spectral lines are not displaced.

But if the mass is in motion at right angles to the wave of light, as indicated by the arrow, the front of the wave will simi-

[1] A similar paper, in Latin, is published in the *Memorie della Pontifica Accademia dei Nuovi Lincei*, **21**, 1903.

larly emerge into the ether at a, and will also similarly have reached a' when its rear arrives at a; but the rear will not yet have emerged at a into the ether, since it must travel farther, to b', in order there to overtake the boundary of the mass which is in the meantime intervening. While this distance is traversed in the mass the front of the wave must pass beyond a' over the distance $ab'n$ in ether. We therefore have for the wave-length in ether:

for mass at rest,

$$a'b' + b'a = \lambda_0 ,$$

for mass in motion,

$$a'b' + b'an = \lambda_1 ,$$

whence

$$\lambda_1 - \lambda_0 = b'a(n-1) . \qquad (1)$$

The distance $b'a$ will be traversed by the intruding mass in the same (apparent) time in which the light passes in ether over the distance $aa' + ab'n$. Therefore if we designate this apparent velocity of the mass by c, that of light in ether by V, we get

$$\frac{ab'}{aa' + ab'n} = \frac{c}{V} , \text{ or } \frac{ab'}{\lambda_1 + ab'} = \frac{c}{V} ,$$

whence we obtain

$$ab' = \frac{c}{V-c}\lambda_1 .$$

Substitution of this value in equation (1) gives the precise formula

$$\lambda_1 - \lambda_0 = \frac{c}{V-c}(n-1)\lambda_1 . \qquad (2)$$

If the mass, acting as a source of light, were to move in the line of sight with a velocity v, the change in the wave-length would be calculated on Doppler's principle with the same precision by the formula

$$\lambda_1 - \lambda_0 = \frac{v}{V-v}\lambda_1 . \qquad (3)$$

For practical computation we may place

$$\lambda_1 - \lambda_0 = \frac{v}{V}\lambda_1 ,$$

where we neglect the velocity of the source as very small compared with that of light. In the case before us we cannot similarly neglect c as compared with V, as the calculation of c leads to enormous values.

We therefore obtain a different alteration of wave-length when the ray passes from the ether into the mass, and not from the mass into the ether as above assumed. In this case the alteration of the wave-length, under otherwise equal conditions, is determined by the formula

$$\lambda_1 - \lambda_0 = \frac{c(n-1)}{V+c}\lambda_0 \ , \tag{4}$$

while in the first case we had, instead of equation (2),

$$\lambda_1 - \lambda_0 = \frac{nc(n-1)}{V-nc}\lambda_0 \ , \tag{5}$$

where the alteration of the wave-length is similarly represented by λ_0.

If the mass intrudes into the path of the ray in the opposite direction, we have only to give the opposite sign to c in the above formulæ.

In order to judge of the applicability of Michelson's explanation to solar phenomena, let us first consider the case represented in Fig. 1 of the emergence of the ray into the ether, which causes the greater displacement of the line.

A comparison of equations (3) and (2) indicates that if the alteration of the wave-length is to be the same according to Doppler as according to Michelson, then we must have

$$\frac{v}{V-v} = \frac{c}{V-c}(n-1) \ . \tag{6}$$

It may be seen from Fig. 1 that

$$c = v \cot \omega \ . \tag{7}$$

On eliminating c from (6) by this equation, we get the equation of condition

$$\tan \omega = \left(1 - \frac{v}{V}\right)\left(n-1\right) + \frac{v}{V} \ .$$

In order to test the validity of Michelson's explanation we must now introduce assumed numerical values into the formula. We will assume at the start that the velocity of the mass, which is to be taken as equal in the two cases, is 300 km per second. This is about the largest velocity that is observed at the time of the maximum solar activity, perhaps once or twice in a year, both in direct ascent and by the displacement of the line according to Doppler's principle. Therefore the intruding mass, according to Michelson's view, is rising with the similar velocity of 300 km at right angles to the emerging ray of light. With this value

$$\tan \omega = 0.999\,(n - 1) + 0.001 \;,$$

and with the omission of the factor 0.999, as it only affects the sixth decimal, which will not be included in our discussion,

$$\tan \omega = (n - 1) + 0.001 \;. \tag{8}$$

The direction of the motion has been wholly left out of consideration above, as it is of no consequence for the purposes of comparison. We will therefore assume that a mass of hydrogen is rising from the Sun into empty space, and that the light emerging from the interior of the mass at the bounding surface inclined to the ray suffers the same change of wave-length as if the mass should move in the line of sight. The following table contains the numerical values for the different quantities computed according to our equation of condition. *D* denotes the density of the hydrogen, *n* its index of refraction referred to a vacuum, and *c* and *ω* are as above:

D $(H=1)$	n	c	ω	Maximum Refraction	Altitude	Duration
		km				secs.
1.5	1.0002	248,000	4′8″	0°48′	..	1
15	1.00208	97,000	10 36	3 42	2″	2
50	1.00695	37,700	27 20	6 44	8	5
100	1.0139	20,000	51 20	9 34	12	10

The fifth column contains the greatest deviation possible for a ray entering into the mass of the given density from the ether at an angle of incidence of 90°. The values permit us to judge

whether the photospheric light could be refracted to the observer by the assumed mass. The sixth column gives the altitude in geocentric seconds ($''$) at which such a refraction could take place. In column 7 is stated the duration of the displacement at that place if the intruding mass shall not occupy a space of over 200,000 km. We notice that even on the assumption of a density of 100 (that of hydrogen being unity) no explanation is possible in this simple manner. The value of *c* comes out so large that in thirty seconds the mass would have to intrude to an enormous expanse, almost equaling the solar radius. It is not permissible to assume a greater density, for even at 100 a hydrogen mass in empty space exhibits a maximum refraction of $9° 34'$. It would therefore necessarily happen occasionally that such a mass would refract the light of the photosphere toward the observer, who would then see a portion of the photosphere at an approximate altitude of $12''$ above the solar limb. Anything of that sort would be easy to see, but has never been observed. This difficulty does not arise at a slighter density, because the altitude resulting would be included within the chromosphere, where the mass would not yet have reached empty space. Hence a displacement lasting for from three to five minutes at the same place, such as is commonly observed, cannot be explained as the result of the elevation of the mass.

The difficulty cited does not occur, however, if we assume that a streamer slightly inclined to the direction of the ray rises with the assumed velocity. The quantity *c* then has no significance. The wave-length of the ray emerging from the interior will be continuously altered by the intrusion of the mass itself; and, as is specially noteworthy, the wave-length will be increased if the streamer is inclined downward with respect to the observer and decreased if it is inclined upward. This explains a displacement of the line toward the red as well as toward the blue, while according to the former assumption the elevation of a compact mass can only have the effect of lengthening the wave. The change of the displacement is also explained without any difficulty, although this offered the greatest obstacles to an explanation on Doppler's principle, for that assumed alterations

of the velocity of from 200 to 300 km during a few minutes, which would predicate quite incredible accelerating or retarding forces. This change of the velocity is explained in an entirely simple manner by the assumption that the streamer does not rise up with precisely the same velocity throughout its whole length —indeed, it would be most remarkable if it should.

An excellent confirmation of Michelson's explanation is also found in the details of observation, since a continuous and considerable variation of the displacements which occur in prominences high above the solar limb is almost always observed, and hardly ever does the displacement remain unchanged for more than a few minutes; while on the explanation by Doppler's principle a uniform displacement evidently ought to be observed, because every motion remains the same unless disturbed by the development of other forces.

A similarly easy and natural explanation is also found for the striking occurrence of opposite displacements in quick succession, particularly in prominences; it is only necessary to assume that the mass is wedge-shaped, with its upper surface inclined backward with respect to the observer, and its lower surface forward.

Another fact, in the best of agreement with this explanation, is that displacements of the spectral lines, particularly at high elevations, as a rule accompany the uprisings of prominences which of themselves are seldom powerful. This fact, however, offers no special difficulty to the explanation on Doppler's principle.

It is, indeed, the fact that I have myself given an entirely different explanation of all these remarkable phenomena, but it is none the less necessary, even if my explanation does correspond with the facts, to yield a place to that of Michelson, according to which the displacements of lines on the Sun, although they cannot all be explained, are nevertheless considerably modified as to their magnitude and significance. The only thing which can be raised against the above explanation according to Michelson would be the smallness of the angle, which can hardly exceed 30′. The bounding surface, therefore, cannot be exactly

plane, but irregularities will only cause the displaced spectral lines to appear diffuse, which is not in contradiction to the observations, as a diffuse character is the more common, particularly where the displaced line seems blurred in the direction of the normal position.

The smallness of the angle offers a more serious difficulty by too greatly limiting the elevation at which the stream appears to the observer. If we assume the density of the ascending stream to be 50, the inclination of the corresponding surface would be $27' 20''$. Therefore, if the place at which the displacement occurs should subtend an angle of $1''$ in elevation for the observer, the streamer would have to be 78,500 km long, that is, about $126''$ geocentric, or 8° on the Sun's limb. This is a very considerable extent, but nevertheless not too great to be assumable, according to observations made elsewhere. One second of elevation is very little, however, and hence the displacements of much greater extent, which are often seen, cannot be explained in this simple manner.

Michelson, however, seems to lay the principal stress on the effect of a prism-shaped mass of high refractory power, which rapidly intrudes into the path of the rays and thus causes a very decided change of the wave-length. He developes a formula in a somewhat complicated manner, from which it follows that if the prism deviates the ray by 60°, the wave-length will be changed by the sudden intrusion as much as if the source of light were to move in the line of sight with the same velocity. From what has been said above we can solve this problem very simply. If a ray of light enters a prism, which is elevated with its edge upward, on one face from the ether into the mass, and on the other face from the mass into the ether, formulæ (5) and (4) will be applicable. It follows from these that the change of the wave-length, though not the same at the two surfaces, is in the same direction, that of increasing the wave-lengths at both contacts. If the edge was downward, the wave-lengths would be diminished in the same degree. Michelson's assumed case with a deviation of 60° is not, however, applicable to the explanation of displacements of lines above the photosphere, in the promi-

nences. We cannot admit that masses of such density rise above the photosphere, because the maximum refraction of 30° due to such masses would necessarily, with corresponding form and position, refract the light of the photosphere toward the observer: a portion of the photosphere would necessarily be seen above the limb, which never has been observed. It is not here disputed that such a refraction can be assumed to occur below the surface of the Sun when vapors of greater refractive power than hydrogen are presumed to be effective.

We can use the above-mentioned action of prismatic masses, however, with great advantage in explaining large local displacements of lines without making abnormal assumptions. Let us regard the above-considered surface of a rising streamer to be, not plane, but covered lengthwise with wave-like structures, which might be roughly treated as prisms with their edges upward. The ray emerging from the interior may then pass through two prisms in order to reach the observer. Equal inclination being assumed, the change of the wave-length would be about five-fold; hence we should get the same displacement as at a simple emergence except with five-fold greater inclination (ω) of the surfaces, or with five times smaller velocity (v) of the mass. It is further known from observations that the large prominences which rise rapidly commonly consist of several parts. Each of these can be regarded as composed of two prismatic portions, one with the edge upward, the other downward. If several portions should happen to lie behind each other for the observer, the light coming from the outside one would have to traverse the intervening rising prismatic masses; the effect of all will be summed up and the same large displacement will be produced without the necessity that the refracting surfaces should have such exceedingly small inclinations. It should, indeed, be remarked that in this case light which has been altered in different degrees is integrated to the observer, so that the displaced light cannot appear separated from the normal line by a clear space. Here the observations also are in good agreement.

We may find an interesting confirmation of this theory in the peculiar appearance of rapidly rising prominences, for they show,

in spite of their extraordinary brilliance, such peculiarly diffuse edges that the observer is prepared by their appearance for the phenomenon of an eruption. This peculiarity is a necessary consequence of the fact that the rising portions must always have more or less strongly inclined lateral surfaces. The inclination must always be greatest at the edges. The displacements will, of course, ordinarily be too small and have too slight an extent to be recognized from the spectral lines. The displacement will, however, be recognized by a diffuse appearance for which the eye is very sensitive; but it is this which has already actually been noticed in observations made hitherto.

We can summarize the results of the above discussion as follows:

1. Michelson's explanation has a sound theoretical basis and is also confirmed by observation.

2. The theory permits a particularly easy explanation of the very peculiar displacements at high elevations of the prominences, but not in every case, so that, while more or less consideration must be given to Michelson's explanation, the hitherto accepted explanation on Doppler's principle cannot be regarded as supplanted.

3. On the assumption that both explanations are based upon the same absolute velocity of the mass, we cannot deny that Michelson's view has the advantage of assuming a vertical direction of motion, while the development of such enormous velocities in the line of sight on the Sun cannot in general be assumed without grave doubts.

In respect to the latter statements I would offer the following evidence: The explanation, on Doppler's principle, of line displacements on the Sun can never be done away with, because the observations prove that such phenomena must be noticed. For occasionally rapidly rising streamers from prominences are observed which are inclined only 30° to the Sun's horizon. If such a streamer is inclined toward the observer and rushes up with a velocity of 300 km, the component in the line of sight will cause a displacement corresponding to a velocity of 260 km. I would cite as an example of this sort the eruption of such a

streamer which was observed by me on June 1, 1900, and incidentally mentioned in the description of the great prominence rising at that time. The streamer appeared in projection inclined 60° toward the horizon, and rose in vertical projection with a velocity of 70 to 80 km. Hence it follows that the horizontal component would be from 35 to 40 km, so that, if this rising streamer had been inclined toward the observer, the corresponding displacement would necessarily have been presented, in an upward direction.

KALOCSA, HUNGARY,
October 4, 1903.

Minor Contributions and Notes.

RECENT SPECTROGRAPHIC OBSERVATIONS OF *NOVAE* WITH THE CROSSLEY REFLECTOR.[1]

THE following observations were obtained with the slitless spectrograph attached to the Crossley reflector:

NOVA AURIGAE.

A negative was obtained on August 29 and 30, 1903, with a total exposure of five hours.

The spectrum shows some decided changes since the previous observations by Stebbins on September 13, 1901.[2]

In 1901 the nebular line at $\lambda 501$ and the lines $\lambda 462$, $\lambda 434$, and $H\delta$ were bright and of about the same intensity. There was also a trace of the band at $\lambda 374$ at that time. *The recent observations fail to disclose any trace of the nebular line at* $\lambda 501$.

Although the plates used in the two cases were of different emulsions, they were of the same make. Other spectrograms taken with these emulsions show them to be about equally sensitive in the region of the chief nebular line.

The lines at $\lambda 462$, $\lambda 434$, and $H\delta$ are about equal to each other in brightness on the recent spectrogram and appear to have decreased in intensity relatively to the continuous spectrum since 1901. There seems to be little change in the band at $\lambda 374$. The magnitude of the *Nova* is now about 14. The 13½ magnitude star at a distance of 2′ from the *Nova*, in position-angle 40°, gives a spectrum closely resembling that of *R Leonis* and similar variables when near maximum. No variations of brightness are shown, however, on the few plates of this region, which have been secured.

NOVA PERSEI.

A spectrogram was secured on July 30, 1903, with an exposure of 2 hours 3 minutes. Striking changes have taken place since the previous observations of October 1901, by Mr. Stebbins, and of January and March 1902 by Mr. Palmer.

[1] *Lick Observatory Bulletin* No. 48. [2] *L. O. Bulletin* No. 35.

The intensities of the lines have decreased very much, relatively to the continuous spectrum, which is now quite strong. $H\beta$ is barely distinguishable, having decreased in intensity materially in the interval. The radiations at $\lambda 434$ appear to have become relatively fainter, while $H\delta$ seems to have suffered only the general decrease in intensity. The greatest change is in the radiations at $\lambda\lambda 339$ and 346. In October 1901 the latter line was the second brightest line in the spectrum, and the former was very strong. The observations of January and March 1902 showed a slight relative weakening in these lines. In the recent observations the line at $\lambda 339$ *has disappeared entirely* and that at $\lambda 346$ is distinguishable only as a very slight brightening in the continuous spectrum, the latter extending to $\lambda 334$. The nebular line at $\lambda 501$ seems to be but little changed in brightness.

A spectrogram obtained with the small slit-spectrograph on February 17, 1903, under rather poor conditions, confirms the great loss of radiations at $\lambda\lambda 339$ and 346. No trace of either of these lines is to be found on that plate, but as the focus is poor in that region, the only conclusion which can be drawn is that these lines could not have been nearly so bright as in 1901 and 1902.

Nova Persei was estimated to be of 11½ or 12 magnitude on July 30.

NOVA GEMINORUM.

Spectrograms of *Nova Geminorum* were secured on August 28 and 31, and on September 2. The last, having an exposure of 1 hour 30 minutes, is the best, but, owing to the great amount of smoke in the air, it is not so dense as the others.

A number of changes in the interval of three and a half months since the observation on May 11 are noticeable. The entire spectrum has grown much fainter. The nebular line at $\lambda 501$ has become relatively stronger, while $H\beta$ has become exceedingly faint. $H\delta$ has also become weak. The line at $\lambda 434$ is now very much the strongest line in the spectrum. Its great strength may be due to the influence of the line at $\lambda 436$, which cannot be separated with the dispersion used. The line at $\lambda 463$ is very much broadened, and is probably composed of several lines, too close for separation with the instrument used. There are also very faint traces of the higher hydrogen lines on the background of continuous spectrum. Owing to the greater transparency of the atmosphere on August 28 and 31, the spectrograms obtained on those nights show a greater density in the ultra-violet, where a faint maximum can be seen in the region of the line at $\lambda 346$.

A careful comparison of the entire series of spectrograms of *Nova Geminorum* obtained with the Crossley reflector discloses some facts in connection with the development of the extreme ultra-violet lines. A number of spectrograms were obtained between April 2 and 8, all of which show a bright maximum in the continuous spectrum, the position of the center of which was found to be at λ350 in two instances and at λ352 in another. The next observation was on April 18. It gives no indications whatever of any maxima in the well-marked continuous spectrum of this region. The next observations, secured on April 26, *show a maximum at λ346, but none in the region of λ350*. The observation of May 11 (the last before interference by the Sun) also shows the maximum at λ346 observed on April 26, but no stronger, and without any trace of the maximum at λ350. The observations of August 28 and 31 show a maximum of similar relative intensity at λ346.

These observations seem to prove conclusively that the line at λ346 made its appearance in the spectrum of *Nova Geminorum* between April 18 and 26, and that just previous to its appearance a brightening, slightly lower in the spectrum, disappeared. No certain indications are to be found of the line at λ339 which was strong in *Nova Persei*.

The apparent absence of the line at λ339 in *Nova Geminorum*, even were it known that there is any relation between it and λ346, would not be significant. In *Nova Persei* λ339 was not nearly as strong as λ346, and even the latter is weak in *Nova Geminorum*.

It would be of great interest to know the history of this ultraviolet region from May 11 to August 28, but unfortunately the Sun interfered and the limited zenith distances at which the Crossley reflector mounting could be used prolonged the interval. On September 2 the seeing was very steady, and the spectrum was examined visually. The nebular line at λ501 was very sharp and distinct, and contained nearly all of the light in the visual spectrum. A line was suspected in the yellow, but could not be confirmed.

The magnitude of *Nova Geminorum* was estimated to be 12 during the recent observations.

The foregoing observations suggest additional facts to be taken into account when considering the physical conditions of the *Novae*.

The line at λ346, which was discovered by Mr. Palmer to exist in some of the nebulæ, appeared in *Nova Geminorum* at about the same as, or a little later than the nebular line at λ501.

The line at λ346 and its companion at λ339, although among the strongest in the spectrum of *Nova Persei*, were the first to disappear.

In *Nova Aurigae*, which has reached a later stage than either *Nova Geminorum* or *Nova Persei*[1] we have observed the extinction of the nebular line at λ 501, since 1901, and a closer approach to the ordinary type of stellar spectrum.

It can scarcely be doubted that the spectra of the three recent *Novae* are destined to attain the same character as those of the great majority of the stars, and as that which Palmer[1] has shown *Nova Cygni* to have reached—*i, e.*, a continuous spectrum without bright lines—and that the whole cycle of changes will occupy but a few years, even in the case of so great an outburst as that of *Nova Persei.*

Mr. Sebastian Albrecht, Fellow in Astronomy in the Lick Observatory, has rendered efficient assistance in making all the recent observations with the Crossley reflector.

C. D. PERRINE.

SEPTEMBER 9, 1903.

VISUAL OBSERVATIONS OF THE SPECTRUM OF *NOVA GEMINORUM* MADE WITH THE THIRTY-SIX INCH REFRACTOR.[2]

THE spectrum of *Nova Geminorum* was observed visually on the mornings of August 17 and 18 with Spectrograph I attached to the thirty-six inch refractor. The three chief nebular lines were well developed, $H\beta$ being faint, λ 4959 somewhat stronger, and λ 5007 relatively very much more intense. There was also a very faint line near λ 4700, but too faint to identify with accuracy. Nothing could be made out either at D or H_a. As far as could be judged, considering the faintness of the object, there was very little continuous spectrum, and the lines were sharp and well defined.

A plate of this region taken on April 6, 1903 (*cf. L. O. Bulletin* No. 37), showed a faint trace of λ 5007, probably just beginning to appear. At present this line is the most conspicuous object in the visual spectrum, and in it is concentrated nearly all the light in this region. The change to the nebular type seems complete.

H. D. CURTIS.

[1] *L. O. Bulletin* No. 35.

[2] From *Lick Observatory Bulletin* No. 48.

ERRATA.

ASTROPHYSICAL JOURNAL, Vol. **18**, October, 1903, in Professor Hartmann's article on "A Revision of Rowland's System of Wavelengths:"

Page 168, ninth line from foot, *for B*, *read A* and *B*.

Page 174, seventh line from top, *for* 5826.582, *read* 5862.582.

Page 182, at head of first column of Table VII, add λ.

Page 185, first line from top, *for* standard, *read* standards.

Page 185, thirteenth line from top, *for* -9, *read* -11.

Page 189, fifteenth line from top, *for* in order to refer them, etc., *read* and referred to the solar spectrum, we have.

Page 189, nineteenth line from top, *for* λ, *read* $\lambda_{\odot}$.